Lecture No. 01 **UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS**

Topic(s) to be covered	Scope of Fluid Mechanics, Definitions of a Fluid

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Scope of Fluid Mechanics ✓ Definitions of a Fluid	

Teaching Learning Material	Student Activity

Lecture Notes

Introduction to Fluids:

Fluide may be defined as a Substance which is capable of Flowing. It has no definite Shape of its own but it Confirms to the shape of the Containing Vessel.

A Small amount of Shear Force exerted on a Fluid will Cause it to undergo a deformation.

Three States:

1. Solid
2. Liquid
3. Gas

Although differences in many aspects liquid and gas have a Common characteristic in which they differ from Solids : The liquid and gas together are called by the Common term "fluids".

Mechanics — Both stationary and moving bodies under the influence of force

- Solid mechanics
- Fluid mechanics [Behavior of Fluids at rest and in motion]
  - Fluid statics [the study of Fluids at rest]
  - Fluid kinematics [Study of Fluids in motion]
  - Fluid dynamics [Study of the effect of Force on the fluid motion]

Scope of Fluid Mechanics:

Fluid Statics
1. Static Fluids
2. Buoyancy
3. Solid moving Fluids

Fluid dynamics

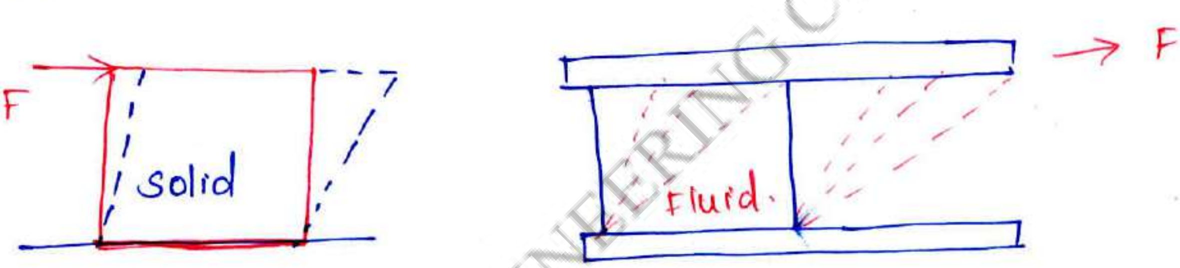
Physical characteristics
1. Continuum: kinetic theory
2. Compressible: - Incompressible
3. Viscous: Inviscid (potential)
   - ✓ Boundary layer theory
   - ✓ Lubrication
   - ✓ Turbulence

Solution method
1. Analytical FM
2. Experimental FM
3. Computational FM

Engineering application
1. Aerodynamics
2. Hydraulics
3. Hydrology
4. Metorology.

Scope of Fluid Mechanics:

1. water Supply systems
2. waste water Treatments Facilities
3. Dam Spillways
4. Valves, windmills, Turbines, Pumps
5. Flow meters, Heating & Air - conditioning System.
6. Hydraulic shock absorbers and brakers.
7. wind loading on buidings
8. Air resistance



Derived units of S.I. System:

Units	Symbol
1. Volume →	m^3
2. Area →	m^2
3. density (mass) →	kg/m^3
4. Dischauge →	m^3/s.

5. Wt. density → N/m^3
6. Surface tension → N/m
7. Momentum → $kg \cdot m/s$
8. Velocity → m/s
9. Acceleration → m/s^2
10. Angular Velocity → rad/s
11. Angular acceleration → rad/s^2
12. Force → N
13. Pressure → N/m^2
14. Dynamic Viscosity → Ns/m^2
15. kinematic Viscosity → m^2/s
16. Power → W.
17. Torque, Work, Energy → Joules (or) N-m.
18. Thermal Conductivity. – W/m.k.
19. Freq. — Hz

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No. 02 UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS

Topic(s) to be covered	✓ Methods of Analysis

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Methods of Analysis	understand, Apply.

Teaching Learning Material	Student Activity

Lecture Notes

Fluid statics: is the Study of Fluids at rest.
It involves the analysis of Fluid

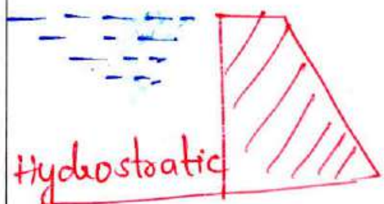
- ✓ pressures
- ✓ Forces
- ✓ moments acting on Fluid System in Static Equilibrium.

Fluid mechanics
(Hydraulic structure)
(Studyon)

Static Condition @ Rest
eg: Dam water etc. tank.

dynamic Condition. @ motion.
eg: pipe flow etc.

water Create Same pressure on dam
↓
Structure



Hydrostatic Condition. down stream. Should be with an std the pressure (resistance)

~~Dynamics~~: Hydrostatic pressure, Pascal's principle and buoyancy Force are all important concept in Fluid Statics.

Fluid mechanics:-
It deals with the behaiver of Fluids at Rest (Static) and at motion condition (dynamics)

Force Cause the Flow :-

open channel Flow :- Gravity cause the Flow

Higher (gravity)

mean

pipe Flow:-
pressure force
Cause the Flow

(Force) pressure.

Dynamics (@ motion condition)

↓ Hydro kinematic (without Considering Force)
To Study the Flow (Force) by Velocity.

↓ Hydro kinetics (with Considering Flow)

Properties of Fluids:-

Solid (molecular Spacing less) ↓ compression.

liquid medium Spacing b/w molecular. Nil.

gas (air) molecular Spacing is large ↑

Fluid

compressible (gas) | In compressible (liquid).

(Same molecular)

Cohesion Force [Cohesive Force) [⟷ Same molecular) ⇓ cohesion Force

↳ It means intermolecular attraction b/w Same liquid (or) Fluid

Solid cohesion ↑ liquid cohesion ↓

Cohesion Force } Solid > liquid > gas (air)

Adhesive Force / Adhesion Force :-

Solid ⟷ liquid (attraction b/w two diff molecular)

It means attraction b/w molecular of a liquid and molecular of the Solid boundary Surface in Contact with a liquid.

Cohesion Force ↓ liquid.

eg: Lotus leaf. have adhesive Force ↑ Cohesion Force ↓.

Method :

1. Pressure measurement:
   1. pressure of a liquid
   2. pressure Head of a liquid
   3. Pascal's Law
   4. Pressure : manometers , Mechanical gauges
      Pressure at a pt In compressible Fluid.

2. Hydrostatic Forces on Surface:
   * Horizontally , Vertically Immersed Surface
   * Inclined , Curved , Dams.

3. Buoyancy and Floatation:
   - Boyancy.
   - Centre of Buoyancy.
   - Types of Equilibrium of Floating Bodies.

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No.	UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS
Topic(s) to be covered	Continuum Hypothesis

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ units ✓ Properties ✓ continuum Hypothesis	

Teaching Learning Material	Student Activity

Lecture Notes

System of units:

1. MKS (System) → metre, Kilogram, Seconds ($1\ kgf = 9.81\ N$)
2. CGS (System) → Centimeter, Gram, Seconds ($1\ N = 10^5$ dyne)
3. SI (System of internation) $1N = kg\ m/s^2$

$1N = 1kg \times 1\ m/s\ /\ s$ (or)

$1N = 1kg \times 1\ m/s^2$

1N → mass = 1 kg, acc = $1\ m/s^2$

$\boxed{1N = 1kg\ m/s^2}$

SI

Mks

Force – N (units)

$\boxed{1\ dyne = 1g\ cm/s^2}$

(Froce unit) in CGS

$1N = 1000 \times 100\ \frac{cm}{s^2} \Rightarrow 1000 \times 100 \times g\ \frac{cm/s^2}{1\ dyne}$

$\boxed{(SI)\ 1N = 10^5 \text{ dyne } (CGS).}$

$1\,kgf = 1\,kg \times 9.81\,m/s^2$

(mks) (mass) (force)

$\boxed{1\,kgf = 9.81\ \underbrace{kg\,m/s^2}_{N}}$ $\because [kg\,m/s^2 = N]$

$1\,kgf = 9.81\,N.$

$1N = kg\,m/s^2$

$1N = ML^{-2}L$

density $\rightarrow kg/m^2$.

Dimension: (SI System)

1. metre $\rightarrow$ L (distance)
2. Second $\rightarrow$ sec $\rightarrow$ T (time)
3. kg $\rightarrow$ m.

Properties:-

1. Mass density (or) density (or) Specific mass (ρ)

density = $\frac{\text{mass}}{\text{Volume}} \left(\frac{m}{V}\right) kg/m^3$ (unit)

fluid $1m^3 \rightarrow kg/m^3$

density Sp. mass of a Fluid is the mass which it per unit volume $\left(\frac{m}{V}\right)$ at a standard temperature and pressure

$$\boxed{(\rho) = \frac{m}{V}}$$

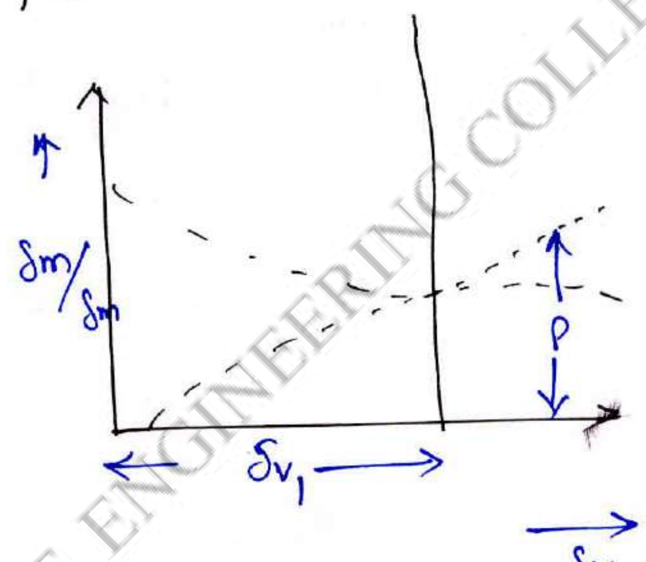
mass density : Fluid will increase with Increasing Pressure

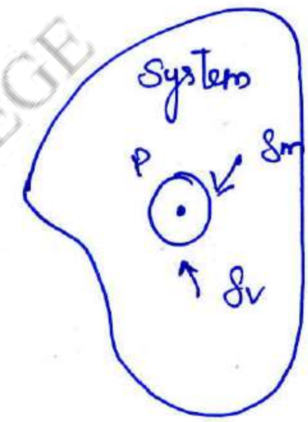
$\boxed{\rho \uparrow \text{ pressure } \uparrow}$ (gass major)

mass density of Fluid decreases with increasing temperature

$\boxed{\text{temperature } \uparrow \ \rho \downarrow}$

Continuum Concept:





All Substance are made up of molecules. molecules inside the Substances are in Constant motion and Collide with each other. In gases, the molecules are not closely Spaced.

So the study of motion of Individual molecule is decribed with the help of Statistical methods.

∴ The fluid is Considered as a Continuous medium called Continuum.

A of mass δm and occupying a volume of δv the density at a point A can be defined as

$$\rho = \lim_{\delta v \to \delta v_c} \left(\frac{\delta m}{\delta v}\right)$$

where δv_c be the Smallest Volume about the point A in which the fluid can be Considered as Continuum.

Suggested Questions / Assignments /Home works / any other

Text Books/ Reference Books

S.No	Title	Author	Publisher
1.			
2.			
3.			

Any other suggested Materials

Lecture No. 4 UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS

Topic(s) to be covered	System and Control Volume Approach.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	System and control volume Approach	

Teaching Learning Material	Student Activity

Lecture Notes

System and Control volume Approach:

System Approach:

A System refers to a Fixed, identifiable quantity of mass which is separated from its Surrounding by its boundaries.

The Boundary Surface may vary with time however no ~~more~~ mass Crosses the System Boundary. In fluid mechanics an Infinitesimal lump of fluid is Considered as a System and is referred as a Fluid elements or a particle.

Since a fluid particle has larger dimension that the limiting volume (refer to section fluid as a continuum).

The continuum concept for the flow analysis is valid.

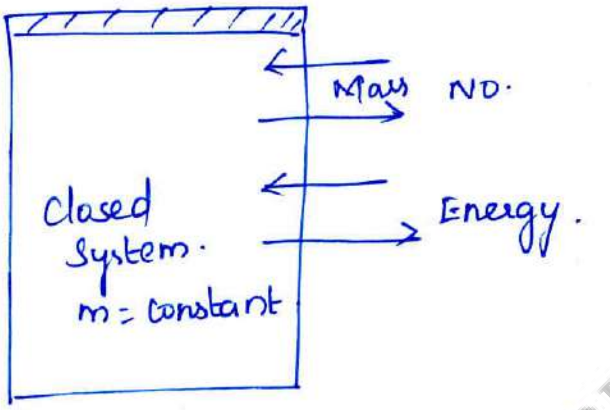
Control Volume:-

The control volume is a fixed, identifiable region in space through which flows.

The boundary of the control volume is called control surface.

The fluid mass in a control volume may vary with time. The shape and size of the control volume may be arbitrary.

Systems may be Considered be closed (or) open, depending on whether a fixed mass or fixed volume in Space is chosen for study.



A Colosed System (also known as a Control mass) Consists of a fixed amount of mass and no mass can cross its boundary. That is no mass can enter or leaves a closed System.

But energy in the form of heat (or) work can cross the boundary, and the volume of a Control System does not have to be fixed.

If, as a Special case, even energy is not allowed to cross the boundary, that System is called an isolated System.

System	Control volume.
* Some fluid property of Fluid described in Space.	Volume Occupying a Space and have a Shape.
* Separated from its Surroun-dings by Boundaries.	Volume Consists of Surface Called Control Surface.
* The particles inside the System will be Same throughout.	The fluid particles inside will Continuously Change.

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No. 5 UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS

Topic(s) to be covered	Reynold's Transporatation theorem.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Reynold's transport theorem.	

Teaching Learning Material	Student Activity

Lecture Notes

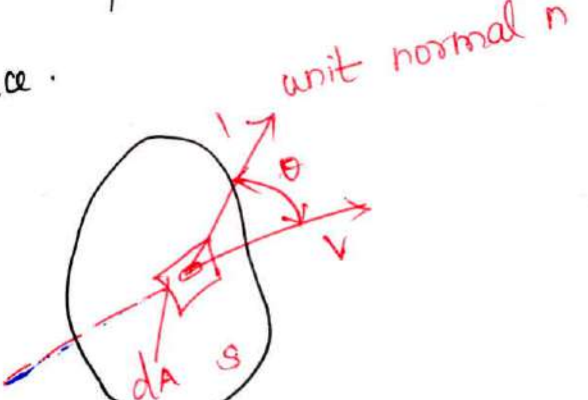
Reynold's transport theorem:-

The Reynold's transport theorem is basically this: that the rate of change of blank in an area is equal to the rate at which the blank inside the area is changing plus the rate at which blank is entering the area.

The Conversation from System analysis to Control Volume analysis is represented by Reynold Transport Theorem.

Volume and mass flow Rate:

Considering an arbitary volume of liquid in Space. It is Separated from its Surroundings by Control Surface.



Elemental area on control Surface Representation

Take a Small element are ΔA on the Control Surface of the Volume. The outward normal of the element area is $\hat{n}$

Let the velocity vector of Fluid passing through the elemental area be $\vec{V}$

$\hat{n}$ & $\vec{V}$ may not be collinear.

The volume of Fluid that will Sweep through the elemental area ΔA in an elementral time Δt.

$$\Delta V = \vec{V}\ \Delta t\ \Delta A \cos\theta$$

$$\Delta V = (\vec{V}\cdot\hat{n})\ \Delta A\ \Delta t$$

$$\frac{\Delta V}{\Delta t} = (\vec{V}\cdot\hat{n})\ \Delta A$$

⇓

Flow rate through the elemental area ΔA.

∴ to get the total volume rate of Flow Q through S. we will first limit the elemental area. ΔA

$$\Rightarrow \lim_{\Delta t \to 0,\ \Delta A \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$\boxed{Q = \int_S \frac{dv}{dt} = \int_S (\vec{v} \cdot \hat{n})\, dA}$$

where
Q = total volume flow rate.

If the Fluid Concerned in the Control volume has a density ρ. mass flow rate.

$$\boxed{m = \int_S \rho\, (\vec{v} \cdot \hat{n})\, dA}$$

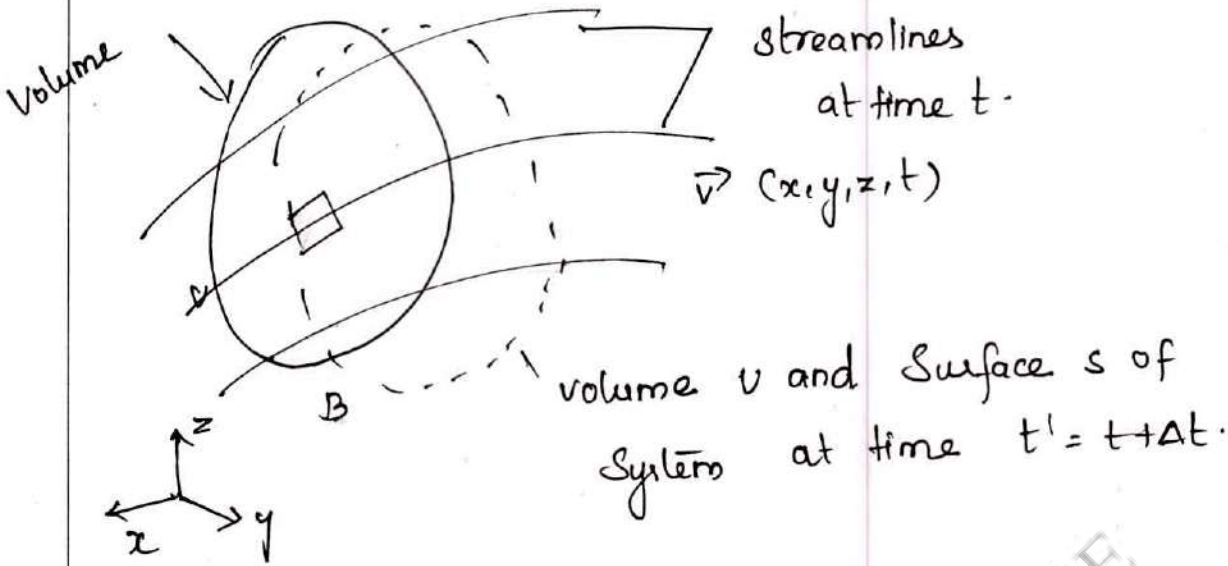
Extensive and Intensive property.

For the Control volume of the Fluid, Let B be any property of the fluid that is related to mass, (eg mass, momentum, energy etc)

Similarly, we can define another property.

$\beta = \frac{dB}{dm}$ (amount of B per unit mass in any element of the fluid)

where β is the Intensive property.



$$\frac{dB}{dt} = \frac{d}{dt}\left[\int_{CV} \beta \rho \, dv\right] + \int_{CS} \left[\beta \rho (\vec{v} \cdot \hat{n}) dA\right].$$

∴ The above equation is Reynolds Transport theorem.

Suggested Questions / Assignments /Home works / any other

Text Books/ Reference Books

S.No	Title	Author	Publisher
1.			
2.			
3.			

Any other suggested Materials

Lecture No. 6 UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS

Topic(s) to be covered	PROPERTIES OF FLUIDS

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Properties of Fluids:

1. Mass density (or) density (or) Specific Mass (ρ)

$$(\rho)\ \text{density} = \frac{\text{mass}}{\text{Volume}} \frac{m}{v}\ kg/m^3\ (\text{unit})$$

fluid $1m^3 \rightarrow kg/m^3$.
density

$$\boxed{\rho = \frac{m}{V}} \quad \text{unit} = kg/m^3$$

Density (or) mass density is defined as the mass per unit Volume, mass of the Fluid Contained in $1\,m^3$ Volume. It is denoted by a Greek Symbol ρ (rho)

(1) Specific mass of a Fluid is the mass which it passess per unit volume

water → $\boxed{\begin{array}{l} \rho_w = 1000 kg/m^3 \\ \rho_{air} = 1.23 kg/m^3 \end{array}}$ (constant) @ std pressure and temperature.

Dimension → $\boxed{kg/m^2 \Rightarrow M L^{-3} T^0}$

unit · Dimension.

Note:-

The density of liquid may be consider as constant while the density the of gases, changes with (in comparable) variation of pressure and temperature.

(2) mass density:- Fluid will increases with increasing pressure.

$\boxed{\rho \uparrow \text{ pressure } \uparrow}$ (gas) major.

mass density of fluid decreases with increasing temperature.

$\boxed{\text{temperature} \uparrow \rho \downarrow}$

(3) unit weight: (or) Sp. wt (or) wt density

weight per unit volume. $g = 9.81$

$$\gamma = \rho g$$

$$\gamma_{(\text{unit volume})} = kg/m^3 \quad m/sec^2$$

$$= \frac{1}{m^3} \underbrace{kg \; m/sec^2}_{N}$$

$$\boxed{\gamma = N/m^3 \text{ (or) } kN/m^3}$$

$\boxed{\text{dimension} = ML^{-2}T^{-2}}$ unit weight.

4. Sp. gravity (or) Relative density (S)

Sp. gravity is the ratio of the Fluid density to the Fluid density of the Std ref. Fluid.

$$\boxed{S = \frac{\rho_{liquid}}{\rho_{water\ @\ 40^{\circ}C}}}$$

unit = Nil $\frac{density}{density}$

Dimension = $M^0 L^0 T^0$.

$\boxed{\text{Sp. gravity of water} = 1}$ $\boxed{\text{Sp. gravity of mercury} = 13.6}$

$$\text{Sp. gravity} = \frac{\rho_{mercury}}{\rho_{water}} = 13.6 \times 1000 = \rho_{mercury}.$$

$\boxed{\rho_{mercury} : 136000}$

Sp. gravity$_{(oil)}$ = 1.8

$$\frac{\rho_{oil}}{\rho_{water}} = 1.8$$

$$1.8 \times 1000 = \rho_{oil} \Rightarrow 1800 = \rho_{oil}.$$

$$\boxed{\rho_{oil} = 1800\ kg/m^3}$$

5. Sp. volume :

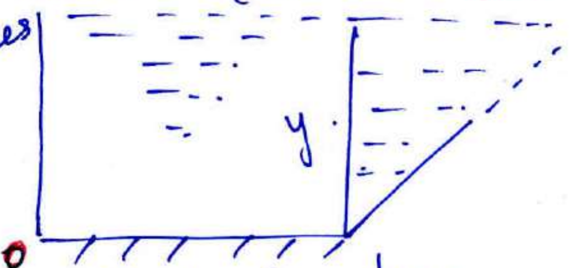
$$\text{Sp. Volume} = \frac{\text{Volume}}{\text{mass}} \left(\frac{m^3}{kg}\right)$$

dimension $\Rightarrow L^3 M^{-1} T^0$.

5. Viscosity

Free Surface (max velocity) Velocity.

↑ Increases



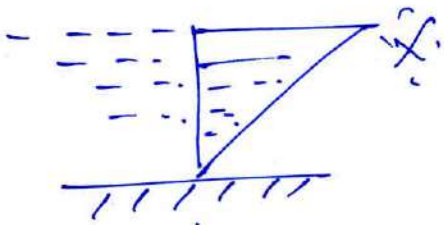
Fixed velocity 0 Rigid Boundary.

Viscosity in

Solid ↑

liquid ↓

Gas ↓.



$\tau = 0$ still

Bed.

$\tau \propto \frac{dv}{dy}$. $\tau = \mu\left(\frac{dv}{dy}\right)$

Shear τ $\frac{dv}{dy} = 0$

depense up velocity.

Still velocity ($\tau : 0$)

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No. 7 UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS

Topic(s) to be covered	✓ Fluid Statics ✓ Manometry.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Measurement of pressure:-

Pressure of a liquid is measured by following devices.

- ✓ manometer
- ✓ Mechanical gauges.

Manometer:-

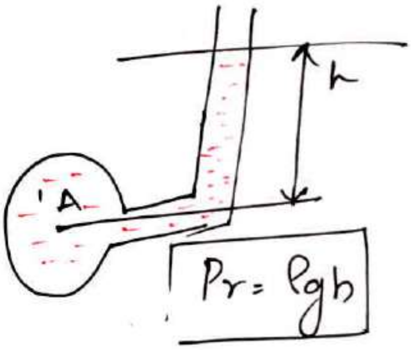
The device is used for measuring the pressure at a point in a fluid by balancing the Volume of Fluid, by the Same or another Column of fluid, they are classified as.

(i) Simple manometer
(ii) differential manometer.

1. Piezometer :- Basic :

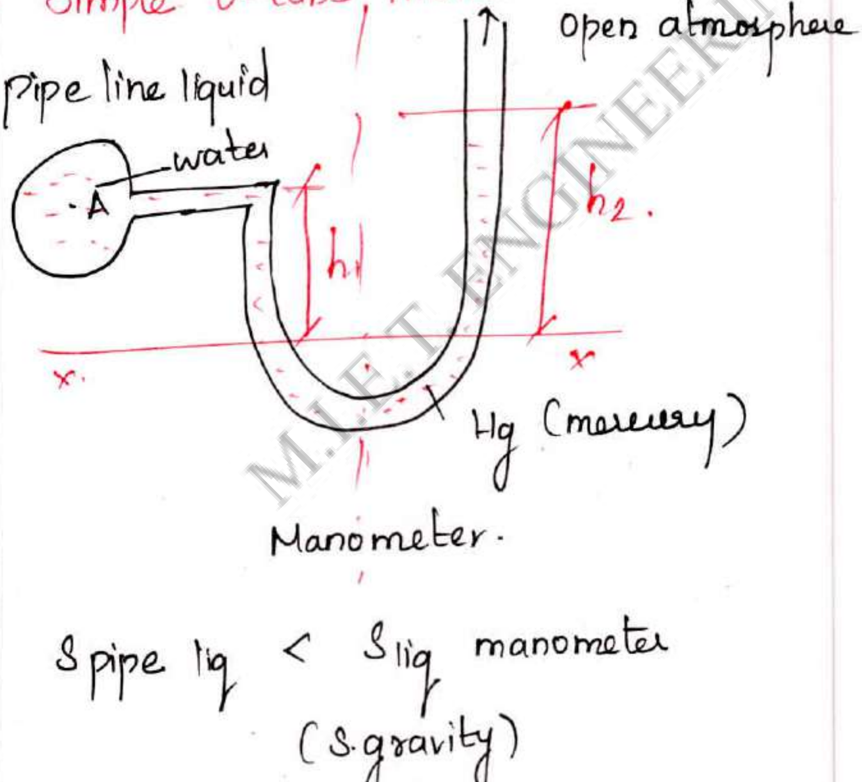
use: Piezometer can be used to measure the pressure @ a pt.

1 atm pressure = 10.3 m



Not having mercury pipe Fluid → tube (manometer) Sample.

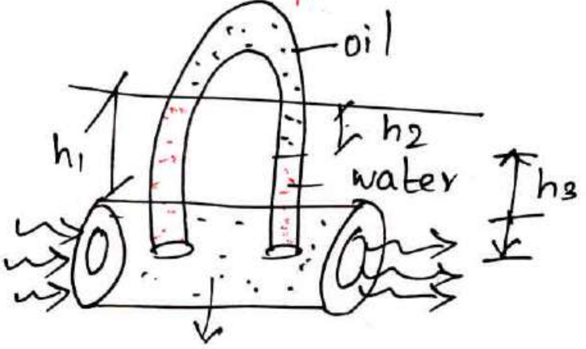
2. Simple U-tube manometer :



Manometer.

S pipe liq < S liq manometer (S.gravity)

4. Inverted differential U-tube manometer :-



$S_{mano\ liq} < S_{pipe\ liq}$

- Negative pressure
- tube line

To measure a Negative pressure

$$H_A - H_S = h_3 - h_2 \; S_2 + h_1 + h_2 S_1$$

$$\boxed{h_A - h_S = h_2 (S_1 - S_2)}$$

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

To measure the +ve pressure the Sp. gravity of manometer Higher then the Sp. gravity of the pipe line

$\boxed{LHS = RHS}$ Eq. Condition.

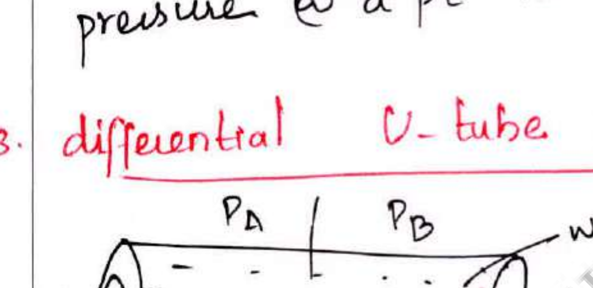
direct method.

To find pressure.

$\boxed{h = \frac{P_r}{W}}$ Sp.wt → water

A simple manometer is used to measure the pressure @ a pt in a pipe line

3. differential U-tube manometer:



Pressure diff b/w two pt in a same pipe line

used to measure the pressure different b/w two points (+ ve pressure)

Spipe liq < S manometer liq. (Specific gravity)

$P_A > P_B$

$h_1 > h_3$

$$\boxed{h_A - h_B = h_2 (S_2 - S_1)}$$

$$P_A - P_B = (h_A - h_B) w w$$

Lecture No. 08	UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS
Topic(s) to be covered	Force on plane and curved Surface.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

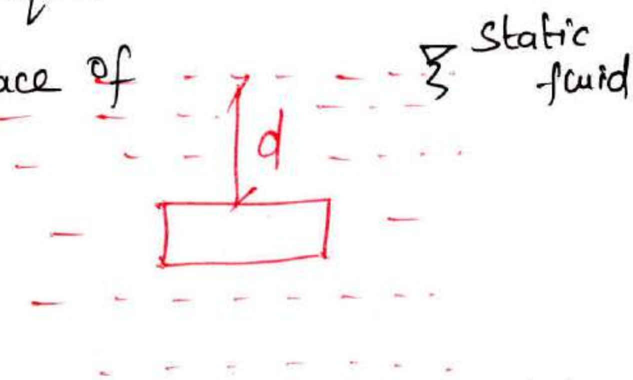
Lecture Notes

Application:-

- ✓ Dam
- ✓ S. Gate watertable.

(i) Hydrastatic Force on the HL plane (free Surface)

For HL plane all point in a place is equal distance (or) equal depth from free Surface of water



$P = \rho g h$

$P = wh$

$P = F/A$

$\boxed{F = pA}$

point of application of total pressure @ Centroid

$\boxed{T = wh.A}$ where F = total pressure

when the Surface of the Structure is Contact with Static Mass of Fluid (water)

the pt of application of total pressure

⇓

Centre of pressure

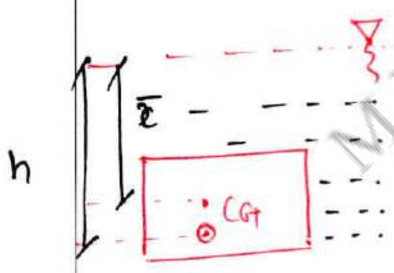
The Centre of pressure to the plane is Centroid of the plane.

$\boxed{F = w g A \cdot \bar{x}}$ Total pressure Force acting on the Surface is Normal (or) ⊥er direction.

A ⇒ Area of the plane

$\bar{x}$ ⇒ Centroid of the plane from free Surface.

Case (ii) Hydrostatic force on the Vertical plane :-



Normal (or) ⊥er

✓ CG acting ⇒ below Surface

✓ for VL plane the Centre of pressure below the CG

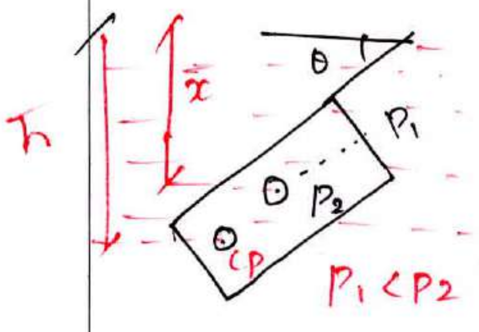
Centre of pressure.

✓ Centre of pressure more where the pressure High

$$\boxed{\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}}$$

✓ $\bar{h}$ = position of Centre of pressure for VL plane

Case (iii) H·F on the inclined plane :-



✓ Centre of pressure acting one plane is below the CG.

$$\boxed{\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x} \Rightarrow cp}$$

Note:

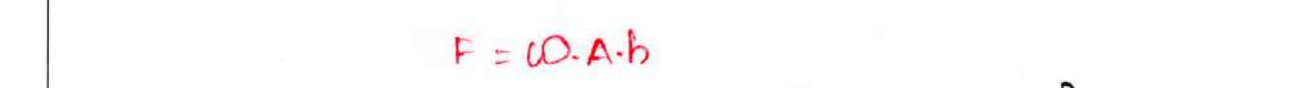
$\theta = 0^\circ \Rightarrow \bar{h} = \bar{x}$ (Horizontal plane)

$\theta = 90^\circ \Rightarrow \bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$ (Vertical plane)

$$\boxed{0^\circ < \theta < 90^\circ \Rightarrow \bar{h} = \frac{I_G \sin^2\theta}{A \cdot \bar{x}} + \bar{x}}$$

Application of Hydrostatic Force:

Dams:-



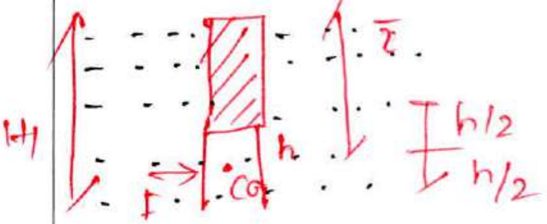
$F = \omega \cdot A \cdot \bar{h}$

$= \omega (1 \times H_1) \times \frac{H_1}{2} = \frac{\omega \cdot H_1^2}{2}$

$$\boxed{F = \frac{\omega \cdot H_1^2}{2}}$$

$$\text{Position } h = \frac{\left(\frac{1 \times H_1}{12}\right)}{(1 \times H_1) \times \frac{H_1}{2}} + H_1/2 = 2/3\, H_1$$

Sluice Gate:



$$F = \omega A\bar{x}$$

$$= \omega \cdot A(H_1 - h/2)$$

$$\text{Position} : \frac{\left(\frac{bh^3}{12}\right)}{(b \times h)(H_1 - h/2)} + (H_1 - h/2)$$

$$= \frac{bh^3}{12} \times \frac{1}{bh(H_1 - h/2)} + (H_1 - h/2)$$

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No. 09 **UNIT I – I FLUIDS PROPERTIES AND FLUID STATICS**

Topic(s) to be covered	Buoyancy and Floatation.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Buoyancy (or) upthrust (↑ Force) ✓ Floatation	

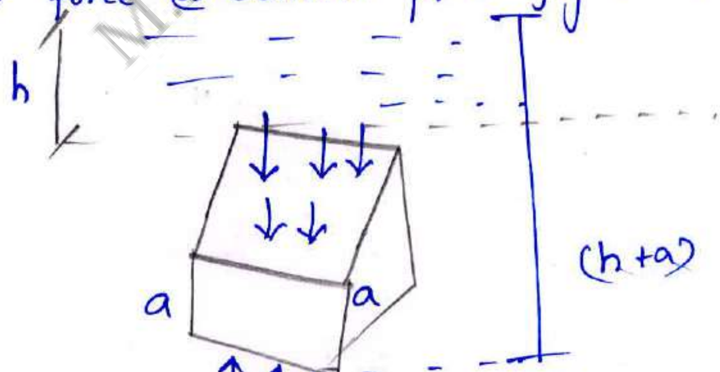
Teaching Learning Material	Student Activity

Lecture Notes

Buoyancy : whenever the object immersed in water (Fluid) there is a upward force ~~ex~~ by water is called as Buoyancy Force

To be in Equilibrium.

Not force @ bottom $pr = \rho g\ (h+a)$



$Pr = \frac{F}{A}$

Floating → metacentre

Submerged → Centre of buoyancy.

$$P_r = \frac{F}{A}$$

(vol object = vol displaced water)

$$A \cdot P_r = F$$

$$\boxed{F = P_r \cdot A}$$

$$w = \frac{wt}{Volume}.$$

Net Force @ bottom $pr = \rho g (h+a)$

$$\rho g (h+a) a^2 - \rho g h \times a^2 = 0$$

$$\rho g \cancel{h a^2} + \rho g a^3 - \rho g \cancel{h a^2} = 0$$

$$\rho g a^3 = 0$$

$\rho g a^3$ = w × volume of object

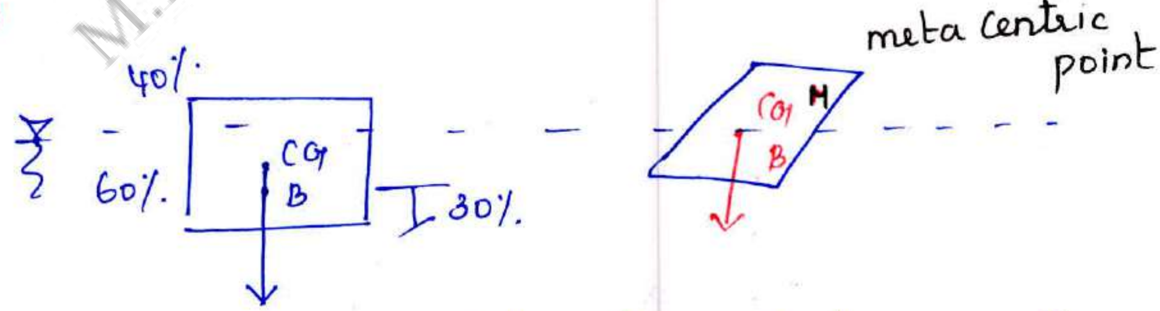
= w × volume of displaced water.

= weight of water displaced.

$$\boxed{\rho g a^3 = \text{Buoyancy Force } (\uparrow)}$$

Buoyancy Force on the Floating Body :-

case (i)

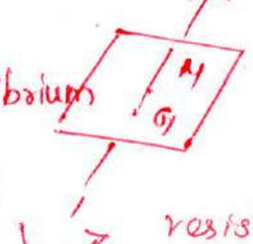


1. The point @ which the intersection of buoyancy force and vertical axis of the object - meta Centre point

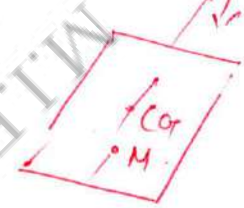
meta Centric point is above the CG of the object means - stable equilibrium:

Restoring couple formed

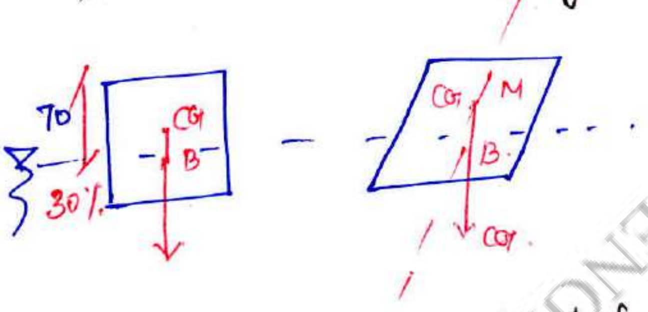
stable equilibrium



distance b/w meta centric pt and CG → meta centric Height.



case (ii)



If the meta Centre pt is below CG leads to

Note:

Case (i)

restoring couple

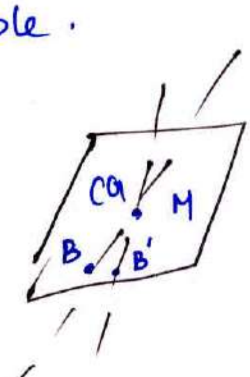
object (opp direction)

(stable)

← meta Centre pt - $\begin{bmatrix} M \\ CG \end{bmatrix}$

Case (ii) $\begin{bmatrix} CG \\ M \end{bmatrix}$ unstable.

Case (iii) Neutral.

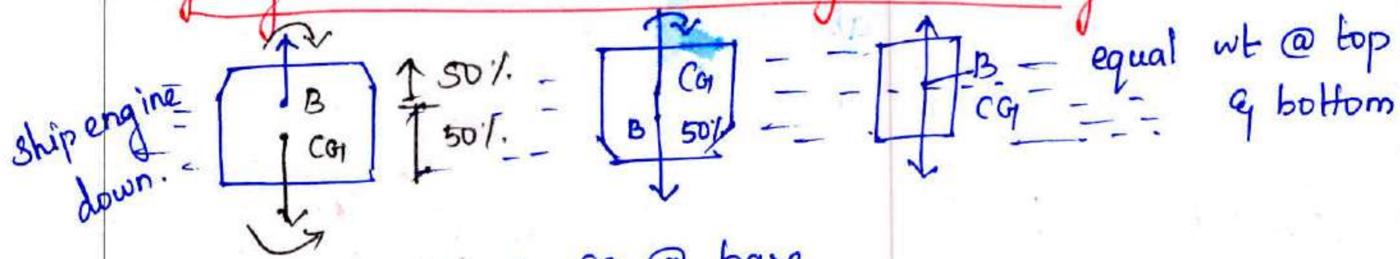


Meta Centre pt coin with CG of the object.

No reaction takes place. If there is any moment on the object.

Neutral equilibrium condition.

Boyancy Force on the Submerged Body :-



- ✓ object Stable → CG @ base
- ✓ Boyancy Force → upward Force sab
- ✓ Boyancy → object depends on Sub merged of the object.

  (100% inside water − 50% ⇒ Boyancy)

- ✓ inside water − move that side.

Suggested Questions / Assignments /Home works / any other

	Text Books/ Reference Books		
S.No	Title	Author	Publisher
1.			
2.			
3.			
Any other suggested Materials			

Lecture No.

UNIT II BASIC CONCEPTS OF FLUID FLOW

Topic(s) to be covered	

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Fluid kinematics:

It is defined as the branch of Science which deals with motion of fluid particles without Considering the force Causing the motion

Types of Flow:-

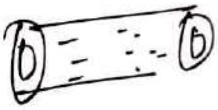
- Steady Flow
- Un Steady Flow.

Steady Flow:-

The Fluid properties such that density, pressure, temperature and velocity does not change with respect to time.

It is mathematically expressed as,

$$\frac{\partial v}{\partial t} = 0 \quad , \quad \frac{\partial p}{\partial t} = 0 \quad , \quad \frac{\partial \rho}{\partial t} = 0$$

 Same Velocity → Flow

D = Same

Q = Constant

Q = a × v

$Q \propto v$

Un steady Flow:

It is the type of Flow in which the Velocity pressure and density at a point Change with respect to time.

It is expressed as

$$\frac{\partial v}{\partial t} \neq 0 \quad , \quad \frac{\partial p}{\partial t} \neq 0 \quad , \quad \frac{\partial \rho}{\partial t} \neq 0$$

2) Uniform & Non - Uniform flow:

Uniform Flow:- It is the flow in which the velocity at any given time does not change with respect to Space (Length of direction of Flow)

$$\frac{\partial v}{\partial s} = 0$$

Non - uniform Flow :-

It is the flow in which the velocity at given time change with respect to space

$$\boxed{\frac{\partial v}{\partial s} \neq 0 \quad \text{Non - uniform Flow.}}$$

3. Laminar and Turbulent flow :-

Laminar Flow:

It is the Flow in which the individual Fluid particles do not cross ~~are~~ one other and move along well - defined paths (or) stream line

It is called as Viscous flow.

Turbulent Flow :- It is flow in which the Fluid particles move in zig - zag way. To Find the type of Flow "Reyold's number" is used.

$$\boxed{Re = \frac{\rho v d}{\mu}}$$

$Re < 2000 \rightarrow$ laminar flow

$Re > 4000 \rightarrow$ Turblent flow.

4) Compressible and incompressible flow :-

In this type of Flow, the density of Fluid change from point to point or density is not constant ~~from~~ for the fluid.

$P \neq$ constant. (In this Flow in which the density is Constant)

5) Rotational (or) Irrotational Flow:-

In this Flow of Fluid particles, by flowing along the Stream line Rotational about their own axis.

⇒ Photo flowing along the Stream line do not rotate about their own accers.

6) Dimensional Flow:-

⇒ $u = f(x, y, z)$ 2 dimensional Flow.

⇒ $v = f(x, y, z)$

⇒ 3D = $w = f(x, y, z)$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 2

UNIT II BASIC CONCEPTS OF FLUID FLOW

Topic(s) to be covered	lines of Flow:

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Stream line ✓ Streak - line ✓ Path - lines	

Teaching Learning Material	Student Activity

Lecture Notes

Path line:-

It is a path Followed by a Single fluid particle is in motion over a period of time

Stream line:-

It is an imaginary line within the flow, So that the tangent at any point on it indicates the velocity at that point

Streak line:-

It is the Curved line which gives a clear picture of the location of the fluid particles which have passed Continuously through a given point.

Stream tube:

A Bundle of neighbouring stream lines may be imagined to from. a passage through which the fluid flows. This passage is known as Stream tube.

Equi - potential line :-

It is a line along which the Velocity pontential function ϕ is Constant.

ϕ = Constant.

Circulation:-

It is defined as the line integral of Velocity field, along a closed path.

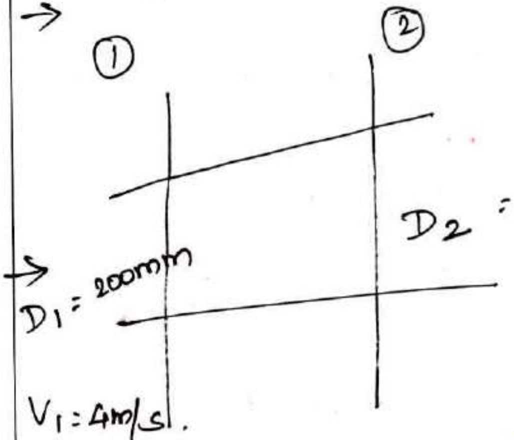
$= \oint_c v. dl. \cos\theta$.

Flownet:-

A grid obtained by drawing a Series of Stream and equipotential line is known as flownet.

1. The dia of pipe at Section ① and ② are 200mm & 300mm respectively. If the velocity of the water flowing through pipe is 4m/s. Find the discharge through the pipe and velocity of water, at section ②.

→



$v_2 = ?$
$Q = ?$

By Continuity equation:

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4}(200)^2 \times 4 = \frac{\pi}{4}(300^2) V_2$$

$$\therefore V_2 = 0.444 \times 4$$

$$\boxed{V_2 = 1.776\ m/s}$$

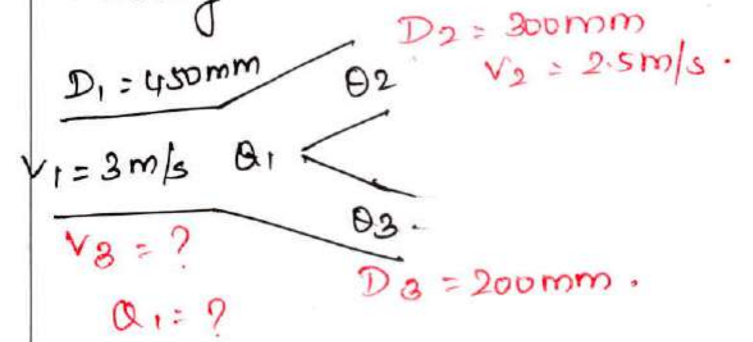
$$Q = A_1 V_1$$

$$= \frac{\pi}{4}(200)^2 \times 4$$

$$\boxed{Q = 125.6 \times 10^3\ mm^2/2}$$

$$Q = 0.125 \times 10^{-3}\ m/s.$$

2. A pipe with 450mm diameter divides into 2 pipes of diameter 300mm & 200mm. It's average velocity in 450mm dia pipe is 3m/s. Find the discharge through 450mm dia pipe and velocity in 200mm dia pipe, if the average velocity in 300mm dia pipe 2.5 m/s.



$Q_1 = Q_2 + Q_3$

$A_1V_1 = A_2V_2 + A_3V_3$

$\frac{\pi}{4}(0.45)^2 \times 3 = \frac{\pi}{4}(0.3)^2 \times 2.5 + \frac{\pi}{4}(0.2)^2 V_3$.

$0.6075 = 0.225 + 0.04 V_3$

$\boxed{V_3 = 9.5625\ m/s}$

$Q_1 = A_1V_1$

$= \frac{\pi}{4}(0.45)^2 \times 3$

$Q_1 = 0.477\ m^3/s$.

$\boxed{Q_1 = 0.477 m^3/s.}$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 3 **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Fluid dynamics:

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

fluid dynamics: It is the Study of behaviour of fluids by Considering the forces Causing the motion.

In the fluid flow, the following forces are present.

(i) Gravity Force
(ii) pressure Force
(iii) Force due to viscosity (or) Viscous force
(iv) Force due to turbulance (zig zag)
(v) Force due to Compressibility.

Velocity potential Function:-

It is defined as a Scalar Function of Space and time Such that its negative derivative with respect to any direction gives the Fluid Velocity in that direction.

Velocity potential $= \phi$

$\phi = f(x,y,z)$ for Steady flow

$$u = -\frac{\partial \phi}{\partial x} \quad v = \frac{\partial \phi}{\partial y} \quad w = -\frac{\partial \phi}{\partial z} \rightarrow (1)$$

where u, v, and w are the Components of Velocity

$$u_r = \frac{\partial \phi}{\partial r} \quad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \rightarrow (2)$$

u_r = velocity component in radial direction

u_θ = Velocity Component in tangential direction.

The Continuity equation for an incompressible Steady Flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.} \rightarrow (3)$$

Two-dimension case : $\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0} \rightarrow (4)$

If any value of ϕ that Satisfies the Laplace equation, will Correspond to Some Case of Fluide Flow.

properties of the potential Flow Function:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

Sub in ① equation:-

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial x}\right) - \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial z}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial z \partial x} + \frac{\partial^2 \phi}{\partial x \partial z}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial y}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial y \partial z} + \frac{\partial^2 \phi}{\partial z \partial y}\right]$$

ϕ, Continuous function, then

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x} ; \quad \frac{\partial^2 \phi}{\partial z \partial x} = \frac{\partial^2 \phi}{\partial x \partial z}$$

$$\boxed{\omega_z = \omega_y = \omega_x = 0.}$$

1. If Velocity potential (ϕ) exists, the Flow Should be Irrotational
2. If Velocity potential (ϕ) Satisfies the

Satisfies the Laplace equation, its represents the possible steady incompressible irrotational Flow.

$$\omega_z = \omega_y = \omega_x = 0.$$

$$\left.\begin{aligned} \frac{\partial^2 \phi}{\partial x \partial y} &= \frac{\partial^2 \phi}{\partial y \partial x} \\ \frac{\partial^2 \phi}{\partial z \partial x} &= \frac{\partial^2 \phi}{\partial x \partial z} \end{aligned}\right\} \quad \phi = \text{continuous function.}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 04 **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Euler's equation of motion along a stream line

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Euler's equation of motion.	

Teaching Learning Material	Student Activity

Lecture Notes

Euler's equation of motion :-

This is equation of motion in which the force due to gravity and pressure are taken into considered.

This is derived by considering the motion of a fluid element along a stream-line as:

Considering a stream line in which flow is taking place in S-direction as shown in fig. Considering a cylindrical element of cross-sections dA and length ds. The force acting on the cylindrical element.

1. Pressure force pdA in the direction of Flow

2. Pressure force

$$\boxed{\left[p + \frac{\partial p}{\partial s} ds\right] dA}$$ opposite to the direction of Flow.

3. wt. of the element $\rho g\, dA \cdot ds$.

Let θ is the angle between the direction of Flow and the line of action of the weight of element

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element $\times$ acceleration in the direction s.

$$\therefore \quad p dA - \left[p + \frac{\partial p}{\partial s} ds\right] dA - \rho g\, dA ds \cos\theta$$

$$= \rho\, dA ds \times a_s \rightarrow (2).$$

Equations of motion:-

according to Newton's Second law of motion the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x- direction

Thus mathematically,

$$\boxed{F_x = ma_x} \rightarrow (1)$$

In the fluid Flow, the following forces are present:-

1. F_g - gravity Force
2. F_p - the pressure force
3. F_v - force due to viscosity
4. F_t - force due to turbulence
5. F_c, force due to Compressibility.

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

(i) If the Force due to Compressibility, F_c is negligible the resulting net force.

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x.$$

equation of motions are called Reynold's equation of motion.

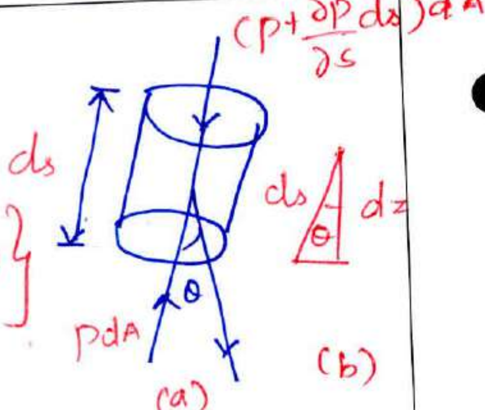
F_t - negligible, the Resulting equation of motion are known as navier Stokes equation.

assumed to be ideal, Viscous force (F_v) is zero and equation of motions are known as Euler's equation of motion.

as - acceleration in the direction of s.

$a_s = \frac{dv}{dt}$ [v is function of s and t)

$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t}$ $\left[\because \frac{ds}{dt} = v\right\}$



If the flow is steady $\frac{\partial v}{\partial t} = 0$

Force on a-fluid equ.

$a_s = \frac{v\,\partial v}{\partial s}$

Sub in as @ ②.

$-\frac{\partial p}{\partial s} ds\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times \frac{\partial v}{\partial s}$

$\div$ by $\rho\,ds\,dA$, $-\frac{\partial p}{\rho\,ds} - g\cos\theta = \frac{v\,\partial v}{\partial s}$

$\frac{\partial p}{\rho\,\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0.$

$\frac{dp}{\rho} + g\,dz + v\,dv = 0$ [euler equation)

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 5 **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Bernoulli's equation.

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Bernoulli's equation.	

Teaching Learning Material	Student Activity

Lecture Notes

Bernoulli's Equation From Euler's Equation:

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

(3) => $\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$

from the Euler's equation (3)

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = 0.}$$

If flow is incompressible, ρ is constant

and

$\therefore \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}.$$

(or)

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}} \rightarrow (4)$$

(4) => Bernoulli's equation in which.

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid (or) pressure head.

$v^2/2g$ = kinetic energy per unit weight (or) kinetic head.

z : potential energy per unit weight (or) potential head.

Assumptions:

The following are the assumptions made in derivation of Bernoulli's equation.

- The fluid is ideal, viscosity is zero
- The flow is steady
- The flow is incompressible
- The flow is irrotational.

Problems:

1. water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0m/s. Find the total head (or) total energy per unit weight of the water at a cross-sectional, which is 5m above the datum line.

→.

diameter of pipe = 5cm = 0.5m

Pressure = p = $29.43 N/cm^2 = 29.43 \times 10^4 N/m^2$

Velocity v = 2.0m/s

datum head z = ~~20~~ 5m

Total head = pressure head + kinetic head + datum head.

$$\text{Pressure head} = \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30m$$

[∵ ρ for water = 1000 kg/m^3]

kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204\,m$

$\therefore$ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z$

$= 30 + 0.204 + 5$

$\boxed{\text{Total head} = 35.204\,m}$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 6 **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Bernoull's equation in Real fluid.

	Lecture Outcome (LO)	Bloom's Level
	At the end of this lecture, students will be able to	

Teaching Learning Material	Student Activity

Lecture Notes

Bernoulli's equation for Real fluid:

The Bernoulli's Equation was derived on the assumption that fluid is inviscid (non-viscous) and ∴ frictionless.

But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration.

Thus the Bernoulli's equation for real fluids between points ① and ②

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_L$$

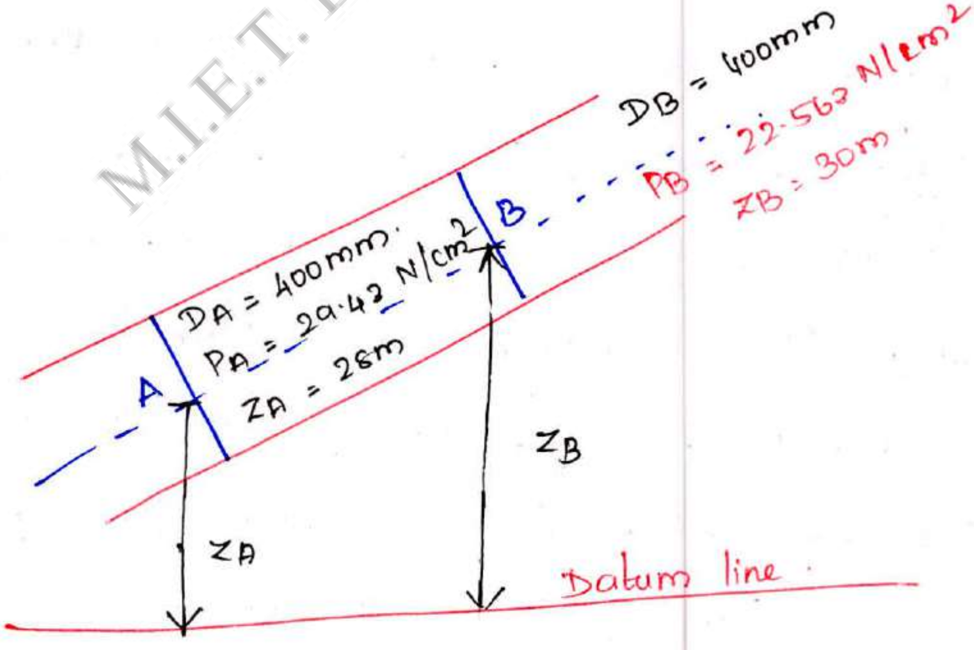
where h_L is loss of energy b/w pt ① and ②

$$\boxed{\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_L}$$

Problem :-

A pipe of diameter 400mm carries water at a velocity of 25m/s. The pressures at the points A and B are given as 29.43 N/cm2 and 22.563 N/cm2 respectively while the datum head at A and B are 28m and 30m. Find the loss of head between A and B.

→



Solution:

→ Dia of pipe, $D = 400mm = 0.4m$

$V = 25 m/s$

at point A, $P_A = 29.43 N/cm^2 = 29.43 \times 10^4 N/m^2$

$z_A = 28m$

$V_A = V = 25 m/s$

∴ Total energy at A.

$$E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28$$

$$= 89.85 m$$

@ point B:-

$P_B = 22.563 N/cm^2$

$= 22.563 \times 10^4 N/m^2$

$z_B = 30m$

$V_B = V = V_A = 25 m/s$

∴ Total energy at B,

$$E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30$$

$$= 23 + 31.85 + 30$$

$$\boxed{E_B = 84.85\, m.}$$

∴ Loss of energy $= E_A - E_B$

$$= 89.85 - 84.85$$

Loss of energy $= 5m.$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 7 **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Linear momentum equation:

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

The momentum equation:-

It is based on the law of Conservation of momentum (or) on the momentum principle, which state that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

The force acting on a fluid mass 'm' is given by the newton's Second law of motion.

$F = m \times a$.

where a is the acceleration acting in the same direction as force F.

But $a = \frac{dv}{dt}$

$$F = m\frac{dv}{dt}$$

$$= \frac{d(mv)}{dt}$$ [m is constant and can be taken inside the differential]

$$\boxed{F = \frac{d(mv)}{dt}}$$ is known as momentum principles. → ①.

① ⇒ can be written as

$$F \cdot dt = d(mv)$$

which is known as the impulse - momentum equation and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

Force exerted by a flowing fluid on a pipe bend.

The impulse - momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Considered two Section (1) and (2)

v_1 = Velocity of flow at section (1).

P_1 = pressure intensity at section (1)

A_1 = area of cross section of pipe at section (1)

v_2, P_2, A_2 = Corresponding values of velocity, pressure and area @ Section (2)

Let F_x and F_y be the components of the Forces by flowing fluid on the bend x and y direction,

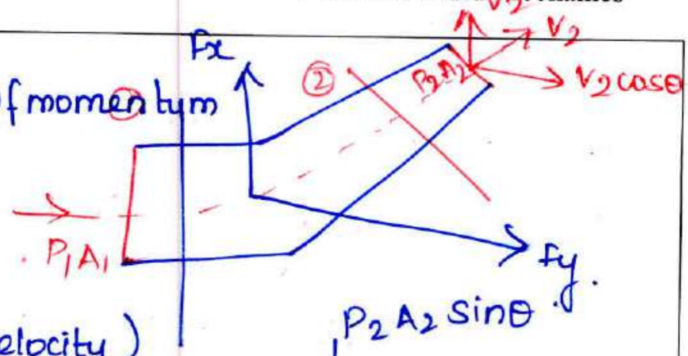
F_x and F_y opp. direction.

x direction = $-F_x$. and in y direction = $-F_y$

P_1A_1 and P_2A_2 on the Section (1) and (2) respectivity.

Net force @ x direction

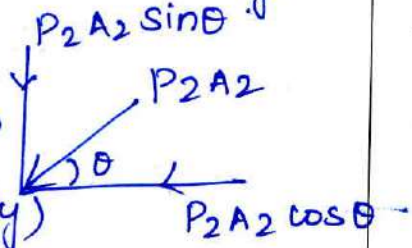
x = Rate of change of momentum



$P_1 A_1 - P_2 A_2 \cos\theta - F_x =$

(mass per Sec) (change of velocity)

= PQ (Final velocity in the direction of x − Initial Velocity)



$= \rho Q (V_2 \cos\theta - V_1)$

$F_x = \rho Q (V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$

IIly $F_y = \rho Q (-V_2 \sin\theta) - P_2 A_2 \sin\theta$

$$\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$$

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$ horizontal direction

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Linear momentum Equation

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Linear momentum equation.	

Teaching Learning Material	Student Activity

Lecture Notes

Linear momentum Equation and Its Application:-

Momentum principle is another very useful principle which leads to the solution of many practical fluid flow problems. In may mechanics problems, it is necessary to determine the force produced on a solid body by the action of flowing fluid.

" The time rate of change of momentum is proportional to the applied force and it takes place in the direction in which the force acts".

It is derived from Newton's Second law of motion.

Newton's Second law of motion,

Force , F = mass x Acceleration = m×a

If the mass of the Fluid is constant,

$$F = m\frac{dv}{dt} \qquad \left[\because a = \frac{dv}{dt}\right]$$

$$F = \frac{d(mv)}{dt}$$

This equation is known as momentum principle.

This equation can be written as

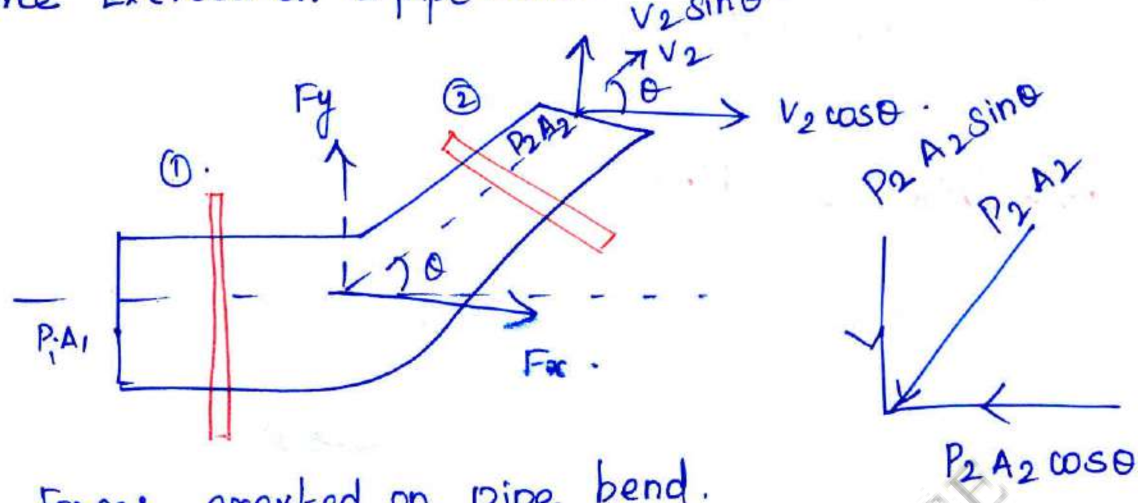
$F \cdot dt = d(mv)$ ⇒ Linear momentum equation

(or) impluse momentum equation.

The momentum principle is applied to the following fluid flow situations.

1. force on pipe bend.
2. Force exerted by a jet striking against a solid surface.
3. Thrust on a propeller
4. Jet propulsion.

1. Force Exerted on a pipe Bend:



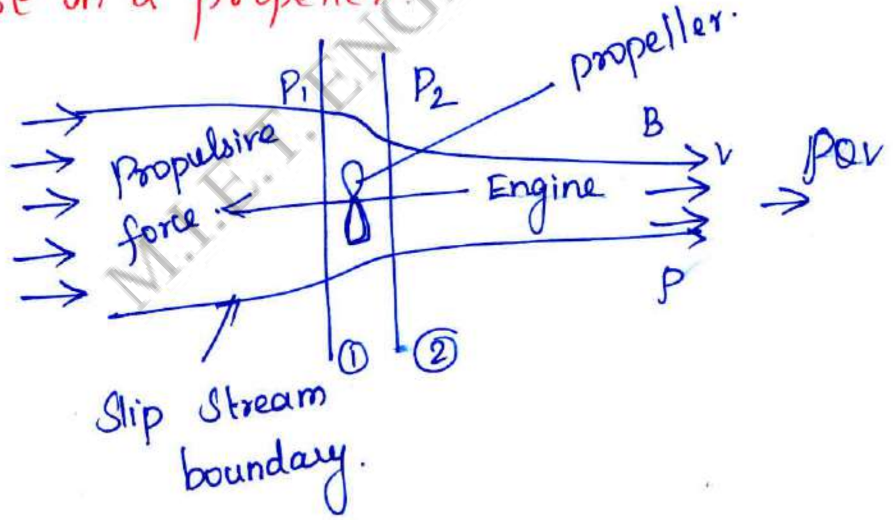
Forces exerted on pipe bend.

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

$$P_1A_1 - F_x = \rho q (v_2 - v_1)$$

Thrust on a propeller:



$$F = \rho Q (v' - v) = \rho A v_p (v' - v)$$

$$\eta_p = \frac{\rho a V_u (v+u)}{\frac{\rho a (v+u)^3}{2}} = \frac{2V_u}{(v+u)^2}$$

Jet propulsion of Ships:-

velocity of Jet relative to the moving jet

$= V+u$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Application to pipe Bends:

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Application to pipe Bends:	

Teaching Learning Material	Student Activity

Lecture Notes

A Bend in pipe line Converging gradually reduces from 700mm to 400mm diameter and deflects the water flow through an angles of 45°. Find the magnitude and direction of the force exerted on the bend if the velocity of flow at 700m Section is 8 m/s and the pressure is 350 kN/m2.

→

Given:-

Inlet diameter, $D_1 = 700mm = 0.7m$

outlet diameter $D_2 = 400mm = 0.4m$

Angle of bend, $\theta = 45^0$

velocity of flow at inlet $V_1 = 8 m/s$

Pressure at inlet $p_1 = 350 \ kN/m^2$

→ Area at inlet, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.7^2 = 0.385\, m^2$

Area at outlet, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.4^2 = 0.126\, m^2$

from Continuity equation,

$$A_1 V_1 = A_2 V_2$$

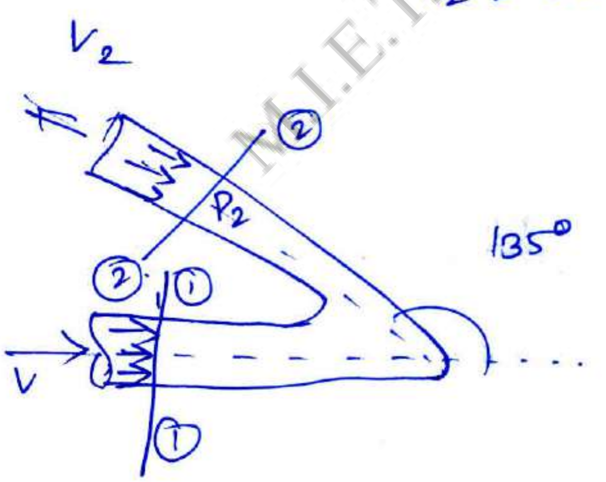
$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.385 \times 8}{0.126} = 24.44\, m/s.$$

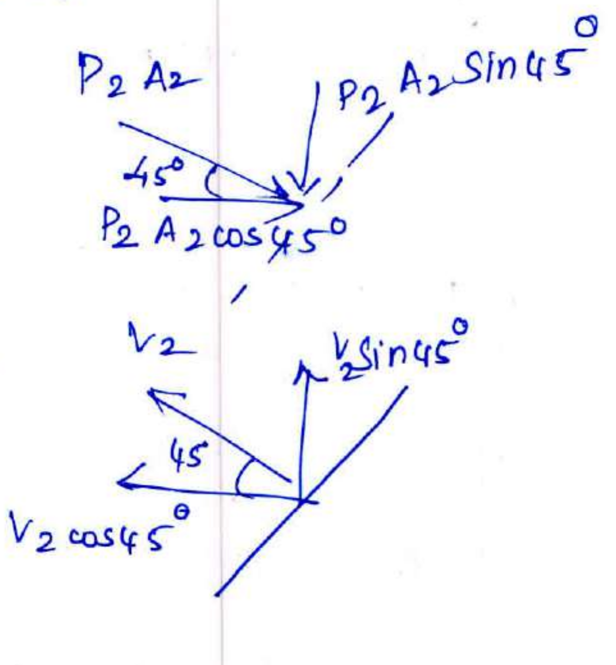
Applying Bernoulli's equation at Section (1) and (2)

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} \qquad [\because Z_1 = Z_2]$$

$$\frac{350}{9.81} + \frac{8^2}{2 \times 9.81} = \frac{P_2}{9.81} + \frac{24.44^2}{2 \times 9.81}$$

$$P_2 = 83.34$$





Dischage $Q = A_1V_1 = 0.385 \times 8 = 3.08\ m^3/s$

$F_x = \rho Q (v_1 - V_2 \cos\theta) - P_2 A_2 \cos\theta + P_1 A_1$

$= [1000 \times 3.08 (8 - 24.44 \times \cos 45^\circ)) - (83.34 \times 10^3 \times 0.126 \times \cos 45^\circ)]$

$+ 350 \times 10^3 \times 0.385$

$F_x = 98737.18 N$

Force along y - direction

$F_y = \rho Q V_2 \sin\theta + P_2 A_2 \sin\theta$

$= (1000 \times 3.08 \times 24.44 \times \sin 45^\circ)$

$+ (83.34 \times 10^3 \times 0.126 \times \sin 45^\circ))$

$= 60652.82 N$

Resultant Force

$F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{98737.18^2 + 60652.82^2}$

$\boxed{F_R = 115878.36 N}$

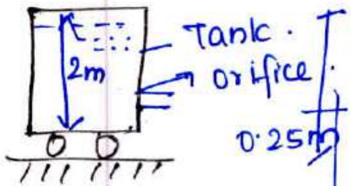
$\tan\theta = \frac{F_y}{F_x}$

$= \frac{60652.82}{98737.18}$

$\theta = \tan^{-1}(0.614)$

$\boxed{\theta = 31.55^\circ}$

A tank 2m high stands on a frictionless trolley and it is full of water. It has an orifice of diameter 120mm at 250mm from the bottom of the tank. If the orifice is suddently opened, what will be the propelling force on the trolley? Assume C_v of the orifice as 0.625.



→ Ht of the tank = 2m

Ht of the orifice from the bottom of the tank = 250mm = 0.25m

Diameter of orifice = d = 120mm = 0.12m

→ Area of orifice $a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.12^2 = 0.0113 m^2$

Head of water = H = 2.0 − 0.25 = 1.75m

$Q = C_d a \sqrt{2gH}$

$= 0.65 \times 0.0113 \sqrt{2 \times 9.81 \times 1.75} = 0.043\ m^3/s$

1.75m

$V = \frac{Q}{a}$

$= \frac{0.043}{0.0113}$

$= 3.81\ m/s$.

$F = Q \rho v$

$= 1000 \times 0.043 \times 3.81$

$= 163.83 N$.

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. **UNIT II BASIC CONCEPTS OF FLUID FLOW**

Topic(s) to be covered	Moment of momentum equation

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	moment of momentum equation.	

Teaching Learning Material	Student Activity

Lecture Notes

moment of momentum Equation:

momentum principle is another very useful principle which leads to the solution of many practical fluid flow problems. In many mechanics problems, It is necessary to determine the force produced on a solid body by the action of fluid flow.

$$F = \text{mass} \times \text{Acceleration} = m \times a$$

If the mass of the fluid is constant

$$F = m\frac{dv}{dt} \qquad \left[\because a = \frac{dv}{dt}\right]$$

$$F = \frac{d(v.m)}{dt}$$

This equation is known as linear momentum equation (or) impulse moment equation.

$\tan\theta = F_y/F_x$ $\quad P_1A_1 - F_x = \rho q (V_2 - V_1)$

(i) Force on pipe bend ~~$F_R = \sqrt{F_x^2+F_y^2}$~~

(ii) Force exerted by a jet of fluid striking against a solid surface $F_R = \sqrt{F_x^2+F_y^2}$

(iii) Thrust on a propeller $\eta_p = \frac{2v}{v+V}$

(iv) Jet propulsion. $\eta_p = \frac{2Vu}{(V+u)^2}$

problems:

A bend in pipeline converging gradually reduces from 700mm to 400mm diameters and deflects the water flow through an angle of 45°. Find the magnitude and direction of the force exerted on the bend if the velocity of flow at 700mm section 8m/s and pressure is 350 kN/m².

→ Given diameter $D_1 = 700mm = 0.7m$

outlet diameter $D_2 = 400mm = 0.4m$

Angle of bend $\Rightarrow \theta = 45^0$

velocity of flow at inlet $V_1 = 8 m/s$.

pressure at inlet $p_1 = 350 kN/m^2$

→ Area at inlet, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.7^2 = 0.385 m^2$

Area of outlet, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.4^2 = 0.126 m^2$

From continuity equation:

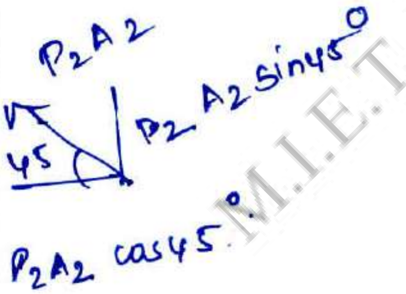
$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{V_2} = \frac{0.385 \times 8}{0.126} \times 24.24 m/s.$$

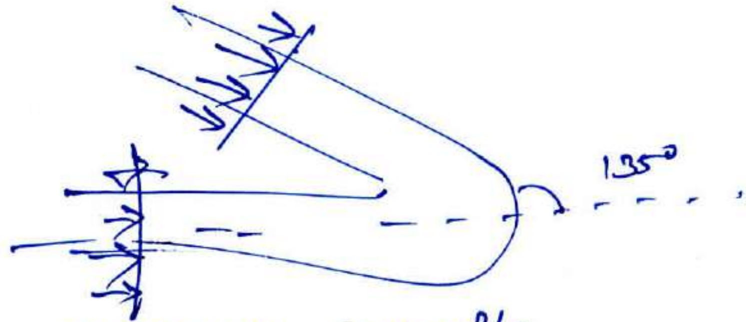
applying Bernoulli's equation at Section ① and ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{350}{9.81} + \frac{8^2}{2 \times 9.81} = \frac{P_2}{9.81} + \frac{24.44^2}{2 \times 9.81}$$



$$P_2 = 83.34 kN/m^2.$$



Dischage $Q = A_1 V_1 = 0.385 \times 8 = 3.08 m^3/s$

Force along x direction (From equation)

$$F_x = \rho Q (V_1 - V_2 \cos\theta) - P_2 A_2 \cos\theta + P_1 A_1$$

$F_x = 98737.18 N.$

Force along y-direction (from equation)

$F_y = \rho Q v_2 \sin\theta + P_2 A_2 \sin\theta = 6065282 N.$

Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{98737.18^2 + 60652.82^2}$

$= 115878.36 N.$

$\tan\theta = \frac{F_y}{F_x}$

$= \frac{60652.82}{98737.18}$

$\theta = \tan^{-1}(0.614) = 31.55^\circ.$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 1 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Fundamental dimensions.	

Teaching Learning Material	Student Activity

Lecture Notes

Unit III Dimensional Analysis and model studies: 7.

Fundamental dimensions - dimensional Homogeneity Rayleigh's method and Buckingham Pi theorem - Dimensionless parameters - Similitude and module studies - distored and undistorted models.

Fundamental dimensions:

Dimensional Analysis:-

Dimensional Analysis deals with the dimensional of physical quantities (length mass, time, Velocity, area, volume etc)

It is a mathematical technique used in research work, for design and for conducting model test.

Length (L), mass (M) and time (T) are the three fixed dimensions which are important in fluid mechanics

Geometric a physical Quantity:-

1. Area $= L^2$ (dimension) unit of area is m^2, so dimension is L^2
2. Volume $= L^3$

unit of Volume is m^3, so dimension is L^3.

Fundamental Physical Quantities:

Length (L), mass (M), time (T)

Kinematic physical Quantities:-

1. Velocity $= \frac{L}{T} = LT^{-1}$
2. Acceleration $= LT^{-2}$ (or) Acceleration due to gravity
3. Discharge $(Q) = L^3T^{-1}$
4. kinematic Viscosity $(\nu) = L^2T^{-1}$
5. Angular acceleration $(\alpha) = T^{-2}$
6. Angular Velocity $(\omega) = T^{-1}$

Dynamic Quantities:-

1. Force → MLT^{-2} (mass × acceleration)

2. Weight → MLT^{-2} (mass × acceleration)

3. Density → ML^{-3}

4. Spec. weight (ω) = $ML^{-2}T^{-2}$

5. Dynamic viscosity (μ) = $ML^{-1}T^{-1}$

6. Pressure intensity (p) = $ML^{-1}T^{-2}$

7. ~~Pressure~~ Modulus of elasticity (k or) E = $ML^{-1}T^{-2}$

8. Power (P) = $ML^{2}T^{-3}$

9. Shear stress (τ) = $ML^{-1}T^{-2}$

10. Torque (T) = $ML^{2}T^{-2}$

① determine the dimension of the Quantities:

(i) discharge (ii) kinematic viscosity (iii) Force

(iv) Sp. weight.

(i) Discharge Q = Area x Velocity

$$= L^2 \times \frac{L}{T}$$

$$\boxed{Q = L^3T^{-1}}$$

(ii) kinematic viscosity $(\nu) = \mu/\rho$

$$\tau = \mu \cdot \frac{du}{dy} \qquad \mu = \tau / du/dy$$

$$\rho = \frac{\text{mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3}.$$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{Force/Area}}{\frac{LT^{-1}}{L}} = \frac{\text{mass} \times \text{Area}}{T^{-1}} = \frac{MLT^{-2}}{L^2 / T^{-1}}$$

$$= MT^{-2}L^{-1}T$$

$$\mu = ML^{-1}T^{-1} \qquad \nu = \mu/\rho = L^2T^{-1}$$

Suggested Questions / Assignments / Home works / any other

S.No	Title	Author	Publisher
	Text Books / Reference Books		
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 2 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Dimensional Homogeneity Rayleigh's method	

Teaching Learning Material	Student Activity

Lecture Notes

Dimensional Homogeneity:-

If the dimensions of each term on both sides of an equation are same, then the equation is known as dimensional homogenity equation.

For example, $V = \sqrt{2gH}$

$V = \sqrt{2gH}$ V = velocity

H_1 = height; g = acceleration due to gravity

$$V = \sqrt{2gH}$$

$$LT^{-1} = \sqrt{LT^{-2} \times L}$$

$$= \sqrt{\frac{L}{T^2} \times L}$$

$$= \sqrt{\frac{L^2}{T^2}} \Rightarrow \frac{L}{T} \Rightarrow LT^{-1} = LT^{-1}$$

$\therefore V = \sqrt{2gH}$ is a dimensional Homogeneity equation.

Method of dimensional Analysis :-

13m, 15m 2016 (i) Rayleight 's method

13m. (ii) Buckingham's π theorem

Rayleight 's method:-

This method is used for determining the expression for a dependent Variable which depends upon maximum 3 or 4 independent Variables only,

Let x is a Variables depends on x_1, x_2, x_3 Variables. According to Rayleight's method, x is a function of x_1, x_2, and x_3 and it can be written as,

$$\boxed{x = f(x_1, x_2, x_3)}$$

This can be written as

$$\boxed{x = k x_1^a, x_2^b, x_3^c}$$

where k is a Constant and a, b, c are the arbitary powers.

Value of a, b and c are attained by Comparing the powers of fundamental dimensions on both Sides.

Thus the expression is obtained for dependent Variable.

1. The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time-period.

Given:

Time period (t) is a function of L and g

Time period (t) → dependent Variables.

L, g → Independent Variable

$$\therefore t = k \cdot L^a g^b \rightarrow ①$$

Substituting the dimensions on both sides,

$$T = kL^a (LT^{-2})^b$$

Equating the powers of length, mass and time (L, M, T) on both sides

Power of T, $1 = -2b$

$$\boxed{b = -1/2}$$

Power of L

$0 = a+b$

$a + (-1/2) = 0$

$$\boxed{a = 1/2}$$

using in ①

$t = k L^{1/2} g^{-1/2}$.

$$\boxed{t = k\sqrt{L/g}}$$

[∴ value of $k = 2\pi$) is determined from (for pendulum) experiment.

$$\boxed{t = 2\pi\sqrt{\frac{L}{g}}}$$

There are an expression for time period is derived.

Home work :-

Find the expression for the power 'P' developed by a pump, when 'P' depends upon the head 'H' The discharge 'Q' and specific weight 'w' of the fluid :-

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 8 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Buckingham's π - Theorem.	

Teaching Learning Material	Student Activity

Lecture Notes

Buckingham's π Theorem:-

It states If there are 'n' Variables (Independent and dependent Variables) is a physical phenomena and if these Variables Contain 'm' fundamental dimensions (MLT) then the Variables are arranged into the dimensionless terms. Each term is Called as π-term.

This theorem is named as the Buckingham's theorem.

Methods of selecting repeating Variables:-

1. The dependent variables Should not be Selected as repeating Variables.

2. The repeating Variables Should be chosen in Such as way that the first Variables Contains geometric

property, Second Variables Contains Flow property and third Variable Contains Fluid property.

Geometric property → length (L), dia (D), Height (H)

Flow property → Velocity (v), Angular Velocity (ω)

Fluid property → Viscosity (μ), density (ρ)

Problems:-

1. The resisting force R of a Super sonic plate during flight can be Considered as dependent upon the length of the aircraft 'l', velocity (v), Viscosity of (air) (μ) Density (ρ) and Bulk modulus of air (k), Express the functional relationship bt these Variables and the resisting force.

<u>Solution</u>:- R, l, v, μ, ρ, k

Total no of Variables, $n = 6$

No of fundamental dimensions $m = 3 (M, L, T)$

$\therefore n - m = 6 - 3 = 3$ dimensionless term.

So three π - terms Should be formed.

$$\pi_1 = l^{a_1}, v^{b_1} \rho^{c_1} R$$

$$\pi_2 = l^{a_2} v^{b_2} \rho^{c_2} \mu$$

$$\pi_3 = l^{a_3} v^{b_3} \rho^{c_3} k$$

First (term π_1)

$$\pi_1 = l^{a_1} v^{b_1} \rho^{c_1} R \rightarrow ①$$

sub sitituting dimensions on b.s

$$M^0L^0T^0 = (L)^{a_1} (LT^{-1})^{b_1} (ML^{-3})^4 (MLT^{-2})$$

Equation powers:

power of M :-

$$0 = c_1 + 1 \Rightarrow \boxed{c_1 = -1}$$

Power of T, $\Rightarrow 0 = -b_1 - 2 \Rightarrow \boxed{b_1 = -2}$

Power of L $\Rightarrow 0 = a_1 + b_1 - 3c_1 + 1$

$$0 = a_1 - 2 - 3(-1) + 1 \Rightarrow a_1 = 2 - 3 - 1$$

$$\boxed{a_1 = -2}$$

using in equation ①

$$\pi_1 = l^{-2} v^{-2} \rho^{-1} R$$

$$\boxed{\pi_1 = \frac{R}{l^2 v^2 \rho^1}}$$

Second π - term:

$$\boxed{\pi_2 = l^{a_2} v^{b_2} \rho^{c_2} \mu \rightarrow ②}$$

Substituting dimensions on both Sides

$$M^0L^0T^0 = (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} (ML^{-1}T^{-1})$$

Equating Power

Power of M, $0 = c_2 + 1 \Rightarrow \boxed{c_2 = -1}$

Power of T, $0 = -b_2 - 1 \quad \boxed{-b_2 = 1}$

Power of L,

$$0 = a_2 + b_2 - 3c_2 - 1$$

$$0 = a_2 - 1 - 3(-1) - 1$$

$$0 = a_2 - 2 + 3 \Rightarrow \boxed{a_2 = -1}$$

using in (2)

$\pi_2 = l^{-1} v^{-1} \rho^{-1} \mu$

$$\boxed{\pi_2 = \mu / l v \rho}$$

Third π-term:

(3) $\Rightarrow$ $\boxed{\pi_3 = l^{a_3} v^{b_3} \rho^{c_3} k}$ → (3)

Substituting the dimensions on bothsides

$M^0 L^0 T^0 = (L)^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1} T^{-2})$

Equating powers

Powers of M, $0 = c_3 + 1$

$$\boxed{c_3 = -1}$$

Power of T,

$0 = -b_3 - 2$

$$\boxed{b_3 = -2}$$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1$

$0 = a_3 - 2 - 3(-1) - 1$

$0 = a_3 - 2 + 3 - 1$

$$\boxed{a_3 = 0}$$

using in (3) $\pi_3 = l^0 v^{-2} \rho^{-1} k$

$$\boxed{\pi_3 = \frac{k}{\rho v^2}}$$

$f(\pi_1, \pi_2, \pi_3) = 0$

$$f\left[\frac{R}{\rho l^2 v^2}, \frac{\mu}{\rho v l}, \frac{k}{\rho v^2}\right] = 0$$

$$\boxed{R = \rho l^2 v^2 \phi\left[\frac{\mu}{\rho v l}, \frac{k}{\rho v^2}\right]}$$

Suggested Questions / Assignments / Home works / any other

The eff. η of a fan, depends on the density ρ, dynamics viscosity μ, of the fluid angular velocity (ω), diameter (D) of the rotar and the dischage (Q). Express interms of dimensionless parameters.

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 4 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	✓ Model Analysis:	

Teaching Learning Material	Student Activity

Lecture Notes

Model Analysis:-

Model : It is the Small Scale replica of a actual Structure (or) machine

The actual Structure / machine is called as prototype

The Study of models of actual machines is called model analysis.

Similitude & types of similarites:-

Similitude:

Similitude is defined as the Similarity b/w the model and its prototypes, which means the model and its prototype, which have Similar properties (or) model and prototypes are Completely Similar.

Types of Similar

There are 3 types of Similarties

(i) Geometric Similarties:

The ratio of all corresponding linear dimension in the model and prototype are equal.

$$\frac{Lp}{Lm} = \frac{bp}{bm} = \frac{Dp}{Dm} = Lr$$

$Lp, bp, Dp \rightarrow$ Length, width and diameter of prototypes

$Lm, bm, Dm \rightarrow$ Length, width & diameter of model

$Lr \rightarrow$ Scale ratio

(ii) Kinematic Similarity:-

The ratio of velocity and acceleration at the corresponding points in the model and the corresponding points in the prototype are same.

$$\boxed{\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r}$$ (For velocity)

where : $V_{p_1}, V_{p_2} \rightarrow$ Velocity at points 1 & 2 in prototypes

$V_{m_1}, V_{m_2} \rightarrow$ Velocity at points 1 & 2 in model

$V_r \rightarrow$ velocity ratio

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r \text{ (for acceleration)}$$

where:

a_{p_1}, a_{p_2} → acceleration at points 1 & 2 in prototypes

a_{m_1}, a_{m_2} → acceleration at points 1 & 2 in model

a_r → acceleration ratio

dynamic Similarity:-

If the ratio's of corresponding forces acting at the corresponding points in the model and prototype are equal then,

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r.$$

where, F_r = Force ratio, F_i = Inertia Force, F_v = Viscous force.

F_g - gravity ratio.

Types of forces acting in moving fluid:-

1. Inertia force (F_i)
2. Viscous force (F_v)
3. Gravity force (F_g)
4. Pressure force (F_p)
5. Surface Tension force (F_s)
6. Elastic force (F_e)

1. Inertia Force : Fi = mass x acceleration

$$\boxed{F_i = \rho A V^2}$$

2. Viscous force:

$$F_v = \tau \times \text{Area}$$

3. Gravity force:-

$$F_g = \text{mass} \times \text{acceleration due to gravity}$$

4. pressure force:-

$$F_p = \text{intensity of pressure} \times \text{area}$$

5. Surface force:-

$$F_s = \sigma \times L$$

6. Elastic force:-

$$F_e = \text{elastic stress} \times A.$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 5 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
✓	Reynold's model law:	

Teaching Learning Material	Student Activity

Lecture Notes

Reynold's model law:-

A pipe of the diameter 1.5m is required to transport an oil of Sp. gravity 0.9 and Viscosity 3×10^{-2} poise at the Rate of 3000l/sec Tests were Conducted on a 15cm diameter pipe using water at 20°c. Find the velocity and rate of flow in the model. Viscosity of water @ 20°c is equal to 0.01 poise.

→

Given:

Dia of prototype, $D_p = 1.5m$

$S = 0.9$

$\mu_p = 3 \times 10^{-2}$ poise

$$\mu_p = 3\times10^{-2} \text{ poise}$$

$$= \frac{3\times10^{-2}}{10} = 30\times10^{-2}\times10^{-1}$$

$$\boxed{\mu_p = 30\times10^{-3} \text{ Ns}/m^2}$$

$$Q_p = 300 \, l/s$$

$$= 300/1000$$

$$\boxed{Q_p = 3 \, m^3/s}$$

model :

$$D_m = 15cm = 0.15m$$

$$\mu_m = 0.01$$

$$\mu_m = 0.01\times10^{-1} \text{ Ns}/m^2$$

$$V_m = ?$$

$$Q_m = ?$$

Solu:

For Reynold's model law:

$$\boxed{\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p v_p D_p}{\mu_p}} \rightarrow (1)$$

ρ_m = Density of water in model $= 1000 \, kg/m^3$

ρ_p = density of oil in prototype $= 0.9 \times 1000 = 900 \, kg/m^3$

Using in ①

$$\frac{1000 \times V_m \times 0.015}{0.01 \times 10^{-1}} = \frac{900 \times V_p \times 1.5}{3 \times 10^{-3}}$$

$$\frac{V_m}{V_p} = 3$$

$$\boxed{V_m = 3 V_p}$$

$$\therefore Q_p = A_p \times V_p$$

$$3 \, m^3/s = \frac{\pi}{4} (D_p^2) \times V_p$$

$$3 = \frac{\pi}{4} (1.5)^2 \times V_p$$

$$\boxed{V_p = 1.697 \, m/s}$$

$$\therefore V_m = 3 \times V_p$$

$$V_m = 3 \times 1.697$$

Velocity of flow in model $\left[V_m = 5.09 \, m/s \right]$

∴ Rate of flow in model

$$(Q_m) = A_m \times V_m$$

$$= \frac{\pi}{4} (D_m)^2 \times V_m$$

$$= \frac{\pi}{4} (0.15)^2 \times 5.09$$

$$\boxed{Q_m = 0.089 \, m^3/s}$$

Suggested Questions / Assignments / Home works / any other
H.W: Froude model law.

	**Text Books / Reference Books**		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 6 **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
✓	Classification of Models	

Teaching Learning Material	Student Activity

Lecture Notes

Classification of models:-

1. Undistorted model
2. Distored model.

Undistorted model.

Undistorted modele are those models which are geometrically Similar to their prototypes (or) if the linear dimensions of the model and its prototypes is Same, the nodel is called undistorted

distorted models

A models is Said to be distorted, if it, is not geometrically Similar to its prototypes (or) if the models are having different Scale ratio for

Horizontal and Vertical dimensions, the models known as distored model.

Advantages of distored model:-

1. The Vertical dimensions of the model can be measured accurately.
2. The Cost of the model can be reduced
3. The turbulent flow in the model Can be maintained

University: Q.

Using buckingham's π Theorem, determine the expression for the velocity through a circular orifice, where μ is the head Causing the flow dia 'D', 'μ' is the Co-eff of Viscosity 'ρ' is the mass density and g is the acceleration due to gravity.

→

v, H, D, μ, ρ, g

No of Variable = 6

No of Fundamental Variables = 3.

$n - m = 6 - 3 = 3$

$$\left.\begin{aligned} \pi_1 &= D^{a_1} \cdot g^{b_1}\, \rho^{c_1} \cdot v \\ \pi_2 &= D^{a_2} \cdot g^{b_2}\, \rho^{c_2} \cdot H \\ \pi_3 &= D^{a_3} \cdot g^{b_3}\, \rho^{c_3} \cdot \mu \end{aligned}\right\} \Rightarrow M^0 L^0 T^0 = (L)^{a_1} \cdot (LT^{-2})^{b_1} (ML^{-3})^{c_1} LT^{-1}$$

P: A ~~72~~ 7.2 m height and 15 m long Spillway discharge 94 m^3/s discharge under a head of 2m If a 1:9 Scale model of the Spillway is to be Constructed, determine model dimension, head over Spillway a force of 7500N, determine the force on the prototype.

Gn:

$h_p = 7.2m$

$L_p = 15m$

$H_p = 2m$

$Q_p = 94 m^3/s$

Scale ratio $L_r = 9$

$F_m = 7500N$

Model dimensions (h_m & L_m)

$\frac{h_p}{h_m} = L_r$, $\frac{L_p}{L_m} = L_r$

$\therefore \frac{7.2}{h_m} = 9$

$\boxed{h_m = 0.8m}$

$\frac{15}{L_m} = 9$

$\boxed{L_m = 1.66m}$

Head over model (H_m)

$\frac{H_p}{H_m} = L_r$

discharge

$$\boxed{\frac{Q_p}{Q_m} = L_r^{2.5}}$$

$$Q_m = \frac{Q_p}{L_r^{2.5}} \Rightarrow \frac{94}{9^{2.5}} = 0.386\ m^3/s.$$

$$\boxed{Q_m = 0.386\ m^3/s}$$

Force on prototype:-

$$\boxed{\frac{F_p}{F_m} = L_r^3}$$

$$F_p = 7500 \times (9)^3$$

$$\boxed{F_p = 5.46 \times 10^6\ N}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. **UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Froude model law:	

Teaching Learning Material	Student Activity

Lecture Notes

Problem:- In 1 in 40 model of Spillway, the velocity and discharge are 2m/s and 2.5 m^3/s. Find the corresponding velocity and discharge in the prototype.

Solu:

$V_m = 2 m/s$

$Q_m = 2.5 m^3/s$

V_p & Q_p = ?

1 in 40 model of Spillway

For velocity,

$$\boxed{\frac{V_p}{V_m} = L_r}$$

Scale rate, $L_r = 40$

$$\therefore \frac{V_p}{2} = 40. \Rightarrow \boxed{V_p = 80 m/s}$$

For discharge,

$$\frac{Q_p}{Q_m} = (L_r)^{2.5}$$

$$\frac{Q_p}{2.5} = (40)^{2.5}$$

$$\boxed{Q_p = 25298.2\,m^3/s}$$

X. X. X.

Dimensionless Numbers

The dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force (or) F_g (or) F_p (or) F_s (or) F_e. These dimensionless numbers are also called non-dimensionless parameters.

1. Reynold's Numbers (Re)

It is defined as the ratio of inertia force and the Viscous ~~form~~ foroce of a fluid.

$$\boxed{Re = \frac{\rho v d}{\mu}}$$

2) Froude's number :- (Fe)

It is defined as the Square root of Ratio of inertia force to the gravity force

$$\boxed{Fe = \sqrt{\frac{Fi}{Fg}}}$$

$$\boxed{Fe = \frac{V}{\sqrt{Lg}}}$$

3. Euler's numbers :- (Eu)

It is define as the Square root of ratio of inertia force to the ~~Surface tension force~~ pressure force.

$$\boxed{\cancel{We = \sqrt{\frac{Fi}{Fs}}}} \qquad \boxed{Eu = \sqrt{\frac{Fi}{Fp}}}$$

4. weber's number (We)

It is defined as the Square root of ratio of inertia force to the Surface tension force

$$\boxed{We = \sqrt{\frac{Fi}{Fs}}}$$

5. Mach's number : (M)

It is defined as the Square root of ratio of inertia force to the elastic force

$$\boxed{M = \sqrt{\frac{F_i}{F_e}}}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 01 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Unit IV Incompressible Viscous flow

Reynolds experiment - Laminar flow in pipes and between parallel plates - Development of laminar and turbulent flow in pipes - Darcy - weisbach equation Moody diagram - Major and minor losses of flow in pipe - Total energy line - Hydraulic grade line - Siphon - pipes in series and parallel - Equivalent pipes.

Reynolds Experiment :-

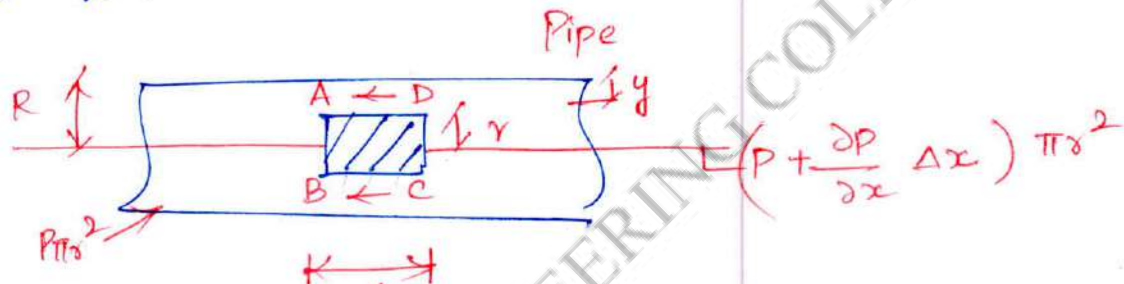
The Reynolds experiment determines the critical Reynolds number of pipe flow at which laminar flow ($Re < 2000$) becomes transitional ($2000 < Re < 4000$) and the transitional ($2000 < Re < 4000$) & the transitional flow becomes turbulent ($Re > 4000$).

Viscous flow:-

The fluid flow in which the particles move smoothly without turbulence like a laminar flow (layer by layer)

Flow through Circular pipe (Hagen - Poiseulli's formula)

Flow through Circular pipe will be Viscous (or) laminar



Consider a flow of Viscous fluid through Circular pipe with radius R. and Consider a fluid element of radius. r.

The length of fluid element will be Δx So, the forces acting in the flow.

1. Pressure force $\rightarrow P\pi r^2$ in the direction of Force.

2. Pressure flow $\rightarrow \left(P+\frac{\partial P}{\partial x}+\Delta x\right)\pi r^2$ opp to direction of flow.

3. Shear force → $\tau \times 2\pi r \Delta r$

Summation of all force in the direction of flow = 0

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$P\pi r^2 - P\pi r^2 - \frac{\partial P}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial P}{\partial x}\cancel{\Delta x}\cancel{\pi} r^{\cancel{2}} = \tau \times 2\cancel{\pi}\cancel{r}\cancel{\Delta x}$$

$$\therefore \frac{-\partial P}{\partial x} \times r = \tau \times 2$$

$$\boxed{\tau = -\frac{\partial P}{\partial x} \times \frac{r}{2}}$$

Velocity distribution (u):-

To obtain Velocity distribution Sub.

$$\tau = \mu \cdot \frac{du}{dy}$$

from diagram,

$$y = R - r$$

$$\therefore dy = -1$$

$$\therefore \boxed{dy = -dr}$$

$$\frac{-\partial P}{\partial x} \cdot \frac{r}{2} = -\mu \frac{du}{dx}.$$

$$\therefore \cancel{-}\frac{du}{dr} = \cancel{-}\frac{\partial P}{\partial x} \cdot \frac{r}{2\mu}.$$

∴ Integrating w.r.t 'r'

$$u = \frac{\partial p}{\partial x} \cdot \frac{r^2}{4\mu} + C$$

$$u = \frac{1}{4\mu} \cdot \frac{\partial P}{\partial x} \cdot r^2 + C \rightarrow (1)$$

From the boundary conditions

$$r = R, \; u = 0$$

$$\therefore 0 = \frac{1}{4\mu} \cdot \frac{\partial P}{\partial x} R^2 + C$$

$$C = \frac{-1}{4\mu} \cdot \frac{\partial P}{\partial x} R^2$$

using in (1)

$$u = \frac{1}{4\mu} \cdot \frac{\partial P}{\partial x} \cdot r^2 - \frac{1}{4\mu} \frac{\partial P}{\partial x} R^2$$

$$u = \frac{1}{4\mu} \frac{\partial \phi}{\partial x} \left[r^2 - R^2 \right]$$

$$u = -\frac{1}{4\mu} \frac{\partial P}{\partial x} \left[R^2 - r^2 \right]$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 2 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

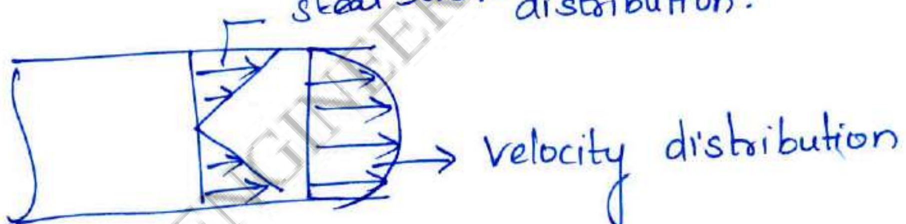
Teaching Learning Material	Student Activity

Lecture Notes

$$\boxed{\frac{P_1 - P_2}{\rho g} = \frac{32 \mu \bar{v} L}{\rho g D^2}}$$

This is Hagen - Poiseuille's formula

Shear Stress & velocity distribution in circular pipe



1. An oil of viscosity 0.1 NS/m2 and relative density 0.9 is flowing through a circular pipe of dia 50mm and of length 300m. The rate of flow of fluid through the pipe is 3.5 l/s. Find the pressure drop in a length of 300m, and also shear stress at the pipe wall and the power require for 300m length of pipe to maintain the flow.

Given:-

$\mu = 0.1 NS/m^2$

$D = 50mm = 0.050m$

$L = 300m$

$Q = 3.5 l/s = 3.5\times10^{-3}$

Pressure drop:-

$$P_1 - P_2 = \frac{32\mu \bar{u} L}{D^2}$$

$$\bar{u} = \frac{Q}{Area}$$

$$= \frac{3.5\times10^{-3}}{\frac{\pi}{4}(0.050)^2} \qquad \bar{u} = 1.785 m/s$$

Sub:

$$P_1 - P_2 = \frac{32\times0.1\times1.785\times300}{(0.050)^2}$$

$$P_1 - P_2 = 685440 \ N/m^2$$

Shearstress at pipe wall:-

$$\tau = -\frac{\partial p}{\partial x}\cdot\frac{R}{2}$$

$$\tau = \frac{-(P_1-P_2)}{(x_1-x_2)}\cdot\frac{R}{2}$$

$$= \frac{P_1-P_2}{x_2-x_1}\cdot\frac{R}{2} \qquad [x_2 - x_1 = 1]$$

$$= \frac{P_1 - P_2}{l} \cdot \frac{R}{2}$$

$$\tau = \frac{685440}{300} \times \frac{0.05/2}{2}$$

$$= 28.56 \, N/m^2$$

Pressure required : $\rho g h_f$.

head loss due to friction, $(h_f) = \frac{32 \mu \bar{u} L}{\rho g D^2}$

$\therefore$ Relative densisty (or) Sp. gravity.

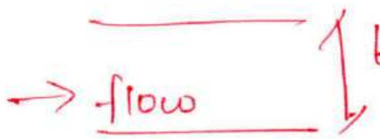
$$= 0.9$$

$$\rho = 0.9 \times 100 = 900 \, kg/m^3$$

Power required $= \rho g Q \times \frac{32 \mu \bar{u} L}{\rho g D^2}$

$$= 2398.8 \, W$$

How to Viscosity ~~fluid~~ find but two Parallel plate ?



max. Velocity $U_{max} = -\frac{1}{8\mu} \frac{\partial P}{\partial x} t^2$

max Shear stress.

$$\tau = -\frac{1}{2} \frac{\partial P}{\partial x} \times t.$$

Average velocity : $\boxed{\bar{u} = -\frac{1}{12\mu} \cdot \frac{\partial P}{\partial x} t^2}$

Ratio of max velocity to Average velocity.

$$\frac{U_{max}}{\bar{u}} = \frac{1 - \frac{1}{8\mu} \cdot \frac{\partial P}{\partial x} \times t^2}{-\frac{1}{12}\mu \frac{\partial P}{\partial x} t^2}$$

$$\frac{U_{max}}{U} = 3/2.$$

Pressure drop:

$$\boxed{P_1 - P_2 = \frac{12\mu \bar{u} L}{t^2}}$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 3 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Flow through pipe :-

Loss of energy in pipe:-

when a fluid is flowing through a pipe, fluid experinces Some resistance due to which Some of the energy of fluid is lost

There are two energy loss.

* Major energy loss
- Minor energy loss.

Major energy loss:

Major energy loss occurs due to friction and it is calculated by

1. Darcy - weisbach formula
2. Chezy's formula

(i) Darcy - weisbach formula:-

$$\boxed{h_f = \frac{4flv^2}{2gd}}$$

h_f = head loss due to friction

f = Co-eff of friction

l = length of a pipe

d = dia of pipe

V = velocity

(ii) Chezy's formula:

$$\boxed{V = C\sqrt{mi}}$$

$c \rightarrow$ Chezy's Constant

$m \rightarrow$ hydraulic mean depth

$i \rightarrow$ Slope

Minor energy loss

minor energy loss occurs due to sudden expansion of pipe and sudden contraction of pipe, and bend in pipe, pipe fittings and obstruction in pipe.

Causes of minor energy loss.

1. loss of head due to Sudden enlargement of pipe:-

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

2. loss of head due to Sudden Contraction :-

$$h_e = 0.5 \frac{V_2^2}{g}$$

3. loss of head at entrance of pipe:-

$$h_i = 0.5 \frac{V^2}{2g}$$

4. loss of head at exit of pipe:-

5. loss of head due to bend in pipe:-

$$h_b = \frac{kV^2}{2g}$$

6 loss of head due to pipe fitting:-

$$h = \frac{kV^2}{2g}$$

$k \Rightarrow$ Co-eff of pipe fitting.

(viii) Head Loss due to obstruction in pipe:-

$$h = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

C_c = co-efficient of contraction.

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 4 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

1. Find the loss of head when the pipe of diameter 200mm is Suddenly enlaged to a dia of 400mm The rate of flow of discharge water through the pipe is 250 l/sec.

Given:-

$D_1 = 200mm$

$D_2 = 400mm$

$Q = 250\ l/s$

$Q_1 = A_1 V_1 \qquad Q_2 = A_2 V_2$

$$V_1 = \frac{Q}{A_1}$$

$$= \frac{0.25}{\frac{\pi}{4}(0.2)^2} = 7.957\ m/s.$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{\frac{\pi}{4}(0.4)^2}$$

$$\boxed{V_2 = 1.989\ m/s}$$

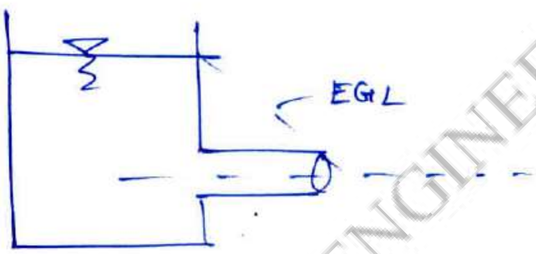
$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.957 - 1.989)^2}{2 \times 9.81}$$

$$\boxed{h_e = 1.815\ m}$$

---

2 mark.



Sketch of HGL & EGL

Hydraulic gradient line (HGL)

The line which given the Sum of pressure head and datum head of a flowing fluid in a pipe with respect to some reference line

$$HGL = \frac{P}{\rho g} + z = \text{Pressure head} + \text{datum head.}$$

Energy line (or) Total energy line:-

The line which gives sum of pressure head, velocity head and datum head of a flowing fluid in pipe with respect to some reference line.

$$\boxed{EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z}$$

Problem: At a sudden enlargement of water main from 240mm to 480mm dia, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Given:-

$D_1 = 240mm$

$D_2 = 480mm$

head loss due to enlargment

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

By Continuity equation:

$$\boxed{A_1V_1 = A_2V_2}$$

$$\boxed{V_1 = \frac{A_2V_2}{A_1}}$$

$$= \frac{\frac{\pi}{4}(D_2)^2 \times V_2}{\pi/4 \, (D_1)^2} = \frac{0.0452 V_2}{0.1809}$$

$$\boxed{V_1 = 4V_2}$$

Sub $\boxed{V_1 = 4V_2}$

$$h_c = \frac{4(V_2 - V_2)^2}{2g} \quad \boxed{h_e = \frac{9V_2^2}{2g}}$$

By Bernoulli's equation :-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} - Z_1\right) = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$\therefore \; 0.10m = \frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g}$$

$$0.10 = \frac{\cancel{6}^{3} V_2^2}{\cancel{2}g}$$

$$0.10 = 3V_2^2$$

$$\boxed{V_2 = 0.18\,m/s}$$

Rate of flow $Q = A_2V_2$

$$= \frac{\pi}{4}(D_2^2) \times 0.18$$

$$= \frac{\pi}{4}(0.48)^2 \times 0.18$$

$$\boxed{Q = 0.0324\,m^3/s}$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

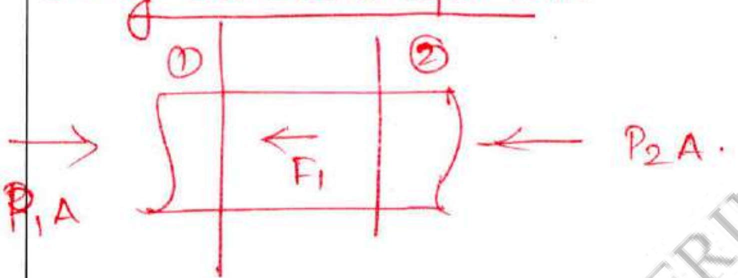
Lecture No. 5 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Darcy weisbach equation:



Consider a uniform horizontal pipe having steady flow and consider two sections ① and ② in pipe. Forces acting on the fluid b/w section ① and ② are.

(i) Pressure force $- P_1 A$

(ii) Pressure force $- P_2 A$

(iii) Frictional force $- F_1$

Resolving all forces in flow direction,

$$P_1 A - P_2 A - F_1 = 0$$

$$A(P_1 - P_2) = F_1$$

F_1 = $\left.\begin{array}{l}\text{Frictional resistance} \\ \text{Per unit wetted area}\end{array}\right\} \times$ wetted area $\times$ Velociting

$F_1 = f' \times (P \times L) \times V^n$ (n - roughness Co-eff

n - 2 for Steady flow)

$$\therefore F_1 = f' \times (P \times L) \times V^2.$$

$$\therefore A(P_1 - P_2) = f' \times (P \times L) \times V^2$$

$$P_1 - P_2 = \frac{f' \times (P \times L) \times V^2}{A} \rightarrow (1)$$

Applying Bernoulli's equation for (1) and (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

Pipe is Horizontal

So $\boxed{z_1 = z_2}$

Diameter is same @ (1) and (2)

$$V_1 = V_2$$

$$\therefore \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\frac{P_1 - P_2}{\rho g} = h_f$$

$$P_1 - P_2 = \rho g h_f \rightarrow (2)$$

Equ (1) and (2)

$$\rho g h_f = \frac{f' \times (P \times L) \times v^2}{A}$$

$$h_f = \frac{f'}{\rho} \times \frac{P}{A} \times \frac{Lv^2}{g} = \frac{f'}{\rho} \times \frac{\pi d}{\frac{\pi d^2}{4}} \times \frac{Lv^2}{g}$$

assume,

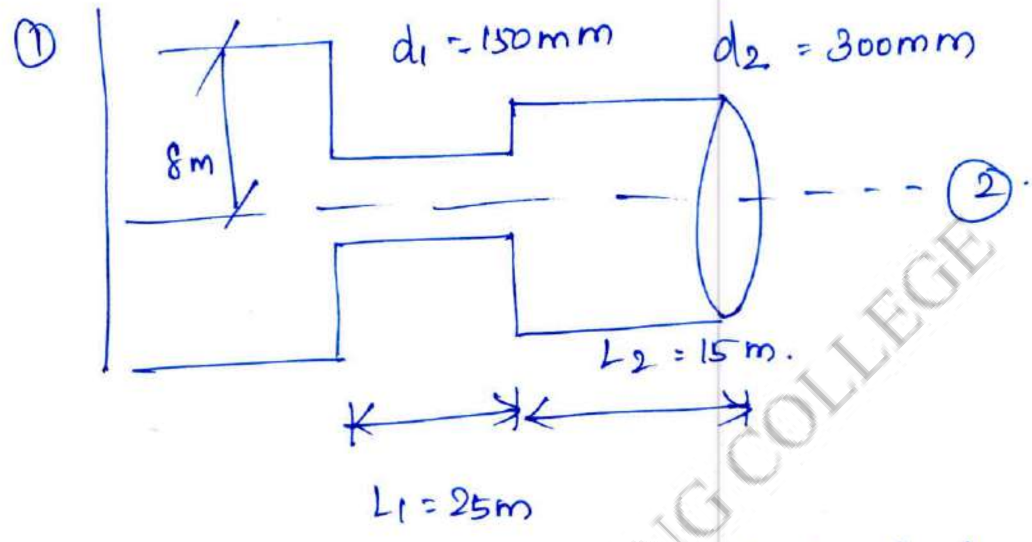
$$\frac{f'}{\rho} = \frac{f}{2}$$

$$\boxed{h_f = \frac{4fLv^2}{2gd}}$$

Hence darcy - weisbach formula:

P, A horizontal pipe line 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end - for the first 25m of its length from the tank, the pipe is 150mm dia & its diameter suddenly enlarge to 300mm. The ht of the water level in tank is 8m above the centre pipe Considering all the losses of head which occur determine all the losses of head which occur, determine the rate of flow. Take $f = 0.01$ for both sections of pipe.

Home work.



Applying Bernoulli's Equation to the tank and Centre of pipe.

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 6 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Flow through pipes in Series (or) Flow through Comp. pipes:	

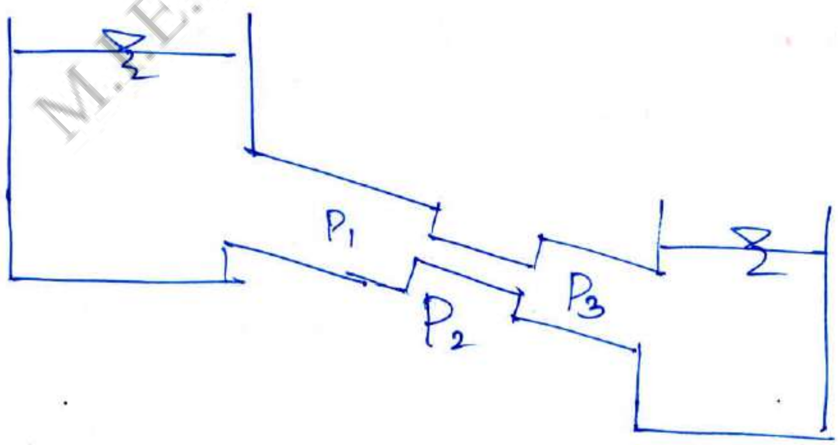
Teaching Learning Material	Student Activity

Lecture Notes

Flow through pipes in Series (or) Flow through Comp pipes:-

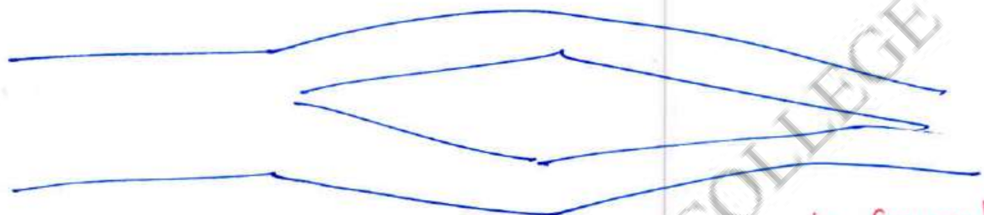
pipes in Series (or) Compound pipes are defined as the pipes of different lengths and different diameters Connected end to end (in Series) to form a pipe line.

Eg:-

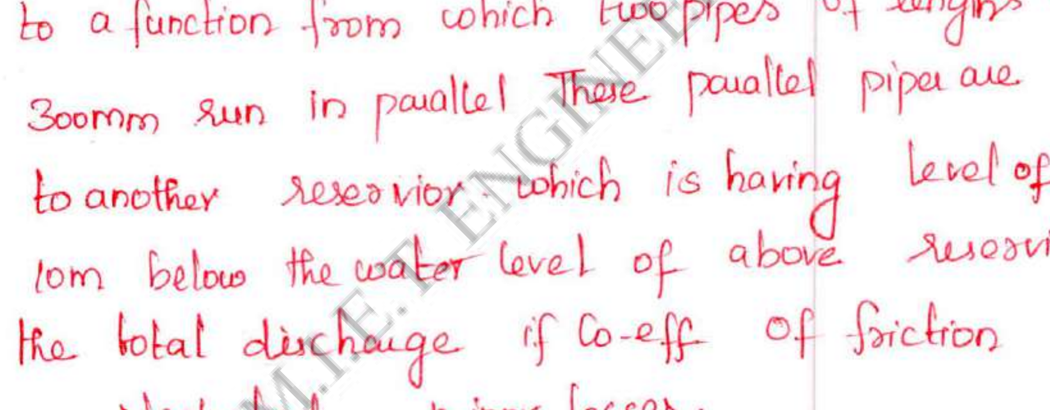


Flow through parallel pipes:-

A main pipe which divides into two or more branches and join together to form a single pipe, then the branch pipes are said to be connected in parallel.



P. A pipe of dia 0.4m and of length 2000m is connected to a reservoir at one end. The other end of the pipe is connected to a junction from which two pipes of lengths 1000m and dia 300mm run in parallel. These parallel pipes are connected to another reservoir which is having level of water 10m below the water level of above reservoir. Determine the total discharge if co-eff of friction f = 0.015. Neglected minor losses.



$f = 0.015$

Length of pipe AB $\rightarrow$ L Length of pipe BD $\rightarrow l_2$

Dia of pipe AB $\rightarrow$ d Dia of pipe (0.3) $\rightarrow d_2$

Length of pipe BC $\rightarrow L_1$ V $\rightarrow$ velocity of pipe AB

Dia of pipe BC $\rightarrow d_1$ $V_1 \rightarrow$ velocity of pipe BC.

Consider the Flow through pipe ABC

Applying Bernoulli's Equation, to the points E and F,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \text{ (in pipe ABC)}$$

$$0+0+10 = 0+0+0+h_f @ AB + h_f @ BC$$

$$10 = \frac{4fLv^2}{2gd} + \frac{4fL_1v_1^2}{2gd_1}$$

$$10 = \frac{4f}{2g}\left[\frac{Lv^2}{d} + \frac{L_1v_1^2}{d_1}\right]$$

$$10 = \frac{4(0.015)}{2 \times 9.81}\left[\frac{2000 \times v^2}{0.4} + \frac{1000 \times v_1^2}{0.3}\right]$$

$$10 = 0.00305\left[5000v^2 + 3333.33v_1^2\right]$$

$$10 = 15.290\,v^2 + 10.19v_1^2$$

discharge through AB = discharge through BC + discharge through BD.

$Q = Q_1 + Q_2$

$Q = A_1 V_1 + A_2 V_2$

$V \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (d_1^2) V_1 + \frac{\pi}{4} (d_2^2) V_2$

$V \cdot \frac{\pi}{4} (0.4)^2 = \frac{\pi}{4} (0.3)^2 V_1 + \frac{\pi}{4} (0.3)^2 V_2$.

$\boxed{V_1 = V_2}$

$V_1 = 0.888 V$

$15.290 V^2 + 8.035 V^2 = 10$

$V^2 (23.325) = 10$.

$V^2 = 0.428$

$\boxed{V = 0.654 \text{ m/s}}$

$Q = A \times V$

$= \frac{\pi}{4} (0.4)^2 \times 0.654$

$Q = 0.0821 \text{ m}^3/\text{s}$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 7 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

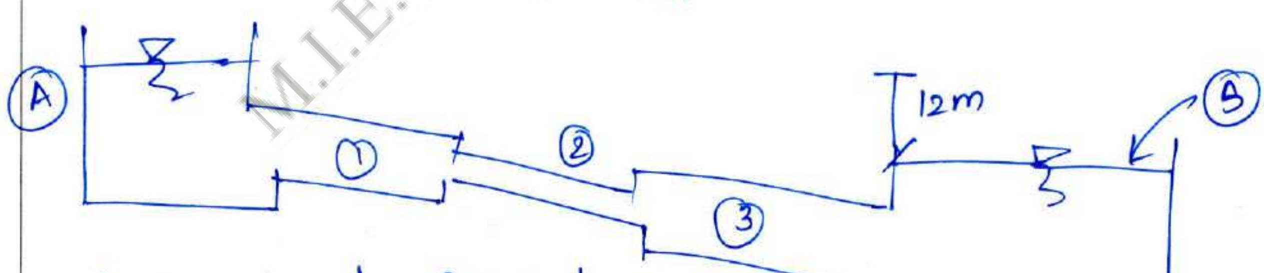
	Lecture Outcome (LO)	Bloom's Level
	At the end of this lecture, students will be able to	
	Problems in Minor losses and Neglecting minor losses.	

Teaching Learning Material	Student Activity

Lecture Notes

Problems: 3 pipes of lengths 300m, 170m and 210m and of diameters 300mm, 200mm and 400mm respectively are connected in series. The difference in water surface levels in two tanks is of 12m. determine the rate of flow of water if the Co-eff of friction are 0.005, 0.0052, and 0.0048

Considering (i) Minor losses
(ii) Neglecting minor losses.



$d_1 = 300mm$ $d_2 = 200m$ $d_3 = 400mm$

$l_1 = 300m$ $l_2 = 170mm$ $l_3 = 210m$

$f_1 = 0.005$ $f_2 = 0.0052$ $f_3 = 0.0048$.

Applying Bernoulli's equation for points (A) and (B)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_i \text{ (entrance)} + h_{f_1} + h_c$$

$$+ h_{f_2} + h_e + h_{f_3} + h_o \text{ (exit)}$$

$$0 + 0 + 12 + 0 + 0 + 0 + \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2}$$

$$+ \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^3}{2gd_3} + \frac{V_3^2}{2g}$$

$$12 = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2 - V_3)^2}{2g}$$

$$+ \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g} \rightarrow (1)$$

By Continuity equation:-

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2}$$

$$= \frac{\pi/4\,(0.3)^2 \times V_1}{\pi/4\,(0.2)^2}$$

$$\boxed{V_2 = 2.25V_1}$$

$$15.290v^2 + 8.035v^2 = 10$$

$$v^2(23.325) = 10$$

$$v^2 = 0.428$$

$$\boxed{v = 0.654 m/s}$$

$$\therefore Q = A \times v$$

$$Q = \frac{\pi}{4}(0.4)^2 \times 0.654$$

$$\boxed{Q = 0.0821 m^3/s}$$

3 ~~pipes~~ of ~~lengths~~ ~~300m, 170m~~ and 210m

~~and~~

Note:-

To Find Neglecting their losses (minor)

$$12 = h_{f_1} + h_{f_2} + h_{f_2}$$

$$12 = \frac{4fL_1v_1^2}{2gd_2} + \frac{4fL_2v_2^2}{2gd_2} + \frac{4fL_3v_3^2}{2g\,d_3}$$

$$v_1 = 1.445 m/s$$

$$Q = Av_1 = \frac{\pi}{4}(0.3)^2 \times 1.445$$

$$Q = 0.102 m^3/s.$$

Home work :-

3 pipes of same length L and D friction factor 'f' are connected in parallel. Determine the diameter of the pipe of length 'L' and fricition factor 'f' which will carry the same discharge for the same head loss use the formula

$$\boxed{h_f = \frac{fLv^2}{2gD}.}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

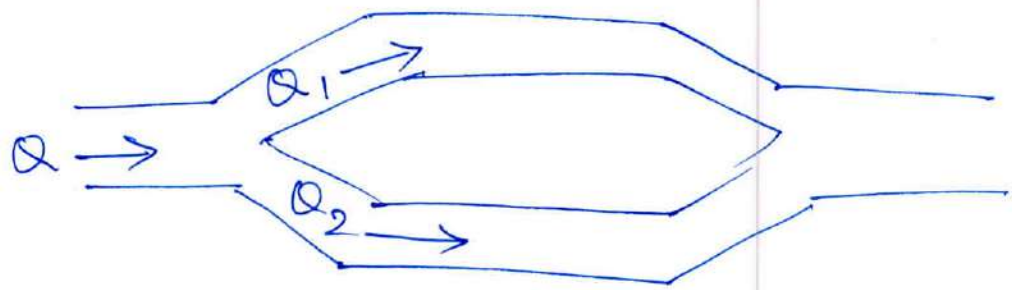
Lecture No. 8 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Flow => parallel pipes.	

Teaching Learning Material	Student Activity

Lecture Notes

A main pipe divided into two parallel pipes which again forms one pipe. The length and diameter of the first parallel pipes are 2000m and 1m, while the length and diameter of the second parallel pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe if the total flow in the main pipe $3 m^2/sec$. Co-eff of friction for each parallel pipe is same and equal to 0.005.



In this arrangement, loss of head for each branch pipe is same.

$$\therefore \boxed{Q = Q_1 + Q_2}$$

loss of head in pipe 1 = Loss of head in pipe 2.

$$\frac{4f_1 L_1 V_1^2}{2gd_1} = \frac{4f_2 L_2 V_2^2}{2gd_2}$$

$$\frac{4\times 0.005 \times 2000 \times V_1^2}{2\times 9.81\times 1} = \frac{4\times 0.005 \times 2000 \times V_2^2}{2\times 9.81 \times 0.8}$$

$$2.038 V_1^2 = 2.548 V_2^2$$

$$\boxed{V_1 = 1.118 V_2}$$

$$Q = Q_1 + Q_2$$

$$3 = \frac{\pi}{4} \times d_1^2 \times V_1 + \frac{\pi}{4} d_2^2 V_2$$

$$3 = \frac{\pi}{4} (1)^2 \times 1.118 V_2 + \frac{\pi}{4} (0.8)^2 \times V_2$$

$$3 = 0.878 V_2 + 0.502 V_2$$

$$\boxed{V_2 = 2.1728 \text{ m/s}}$$

$$\therefore V_1 = 1.118 \times 2.1728$$

$$\boxed{V_1 = 2.42 \text{ m/s}}$$

$$\therefore Q_1 = \frac{\pi}{4} (d_1^2) V_1$$

$$= \frac{\pi}{4} (1)^2 \times 2.42$$

$$\boxed{Q_1 = 1.9 \text{ m}^3/\text{s}}$$

$$Q_2 = \frac{\pi}{4} d_2^2 \times V_2$$

$$= \frac{\pi}{4} (0.8)^2 \times V_2$$

$$Q_2 = 1.108\ m^3/s$$

$$\boxed{Q = Q_1 + Q_2}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 9 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

U.Q. May/June 2014:

An oil of viscosity 0.1 Ns/m² and relative density 0.9 is flowing through a circular pipe of diameter 5cm and the length 300m. The rate of flow of fluid through the pipe is 3.5 liters/sec. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall.

→ ~~Start~~ Solution:-

Given : υ, dynamic viscosity $= 0.1\ Ns/m^2$

Relative Density $S = 0.9$

density of oil $= 0.9 \times 1000 = 900 kg/m^3$

$\because$ density of water $= 1000 kg/m^3$

diameter of pipe $D = 50mm = 0.05m$

Length of pipe $L = 300m$

discharge $Q = 3.5 lit/sec = \frac{3.5}{1000} = 0.0035 m^3/sec$

(i) <u>Pressure drop $(P_1 - P_2)$</u>

$$P_1 - P_2 = \frac{32 \mu \bar{u} L}{D^2}$$

$$\bar{u} = \frac{Q}{Area} = \frac{0.0035}{\frac{\pi D^2}{4}} = \frac{0.0035}{\frac{\pi}{4}(0.05)^2}$$

$$= 1.782 m^3/sec.$$

$$P_1 - P_2 = \frac{32 \times 0.1 \times 1.782 \times 300}{(0.05)^2}$$

$$P_1 - P_2 = 684493.5 N/m^2$$

(ii) <u>Shear stress at the pipe wall (τ_0)</u>

max shear stress $\tau_0 = -\frac{\partial P}{\partial x} \cdot \frac{R}{2}$

Here $-\frac{\partial P}{\partial x} = \frac{(P_1 - P_2)}{x_2 - x_1} = \frac{P_1 - P_2}{L}.$

$$\therefore \tau_0 = \frac{P_1 - P_2}{L} \times \frac{R}{2} = \frac{684493.5}{300} \times \frac{(0.05/2)}{2}$$

$$= 28.52 N/m^2$$

$$\tau_0 = 28.52 N/m^2.$$

(d) At a Sudden enlargement of water main from 240mm to 480mm diameter, the Hydraulic gradient rises by 10mm Estimate the rate of flow. Draw the HGL for the System described.

→ $D_1 = 240mm = 0.24m$

$D_2 = 480mm = 0.48m$

Hydraulic gradient = 10mm

→ $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.24)^2 = 0.0452 m^2$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.48^2 = 0.180 m^2$

Rise of Hydraulic gradient $\left[z_2 + \frac{P_2}{\rho g}\right] - \left[\frac{P_1}{\rho g} + z_1\right] = 10mm$

$= 0.01m$.

rate of flow = Q

Apply bernoulli's equation.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss.}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_2 V_2}{A_1} = \frac{\frac{\pi D_2^2}{4} \times V_2}{\pi/4\, D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2$$

$$h_e = \cancel{4(v_2)}\ \frac{4(v_2 - v_2)^2}{2g} = \frac{3v_2^2}{2g} = \frac{9v_2^2}{2g}$$

$$\frac{P_1}{\rho g} + \frac{4v_2^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \frac{9v_2^2}{2g}$$

But Hydraulic gradient Rise $= \left(\frac{P_2}{\rho g} + z_2\right) - \left(\frac{P_1}{\rho g} + z_1\right) = \frac{1}{100}$

$$\therefore \frac{16v_2^2}{2g} - \frac{v_2^2}{2g} - \frac{9v_2^2}{2g} = \frac{1}{100}$$

$$\frac{6v_2^2}{2g} = \frac{1}{100}$$

$$V_2 = \sqrt{\frac{2 \times 9.81}{6 \times 100}} = 0.181\,m/s$$

dischage :

$$Q = A_2 \times V_2$$

$$Q = \frac{\pi}{4}(0.48)^2 \times 0.181 = 0.03275\,m^3/s$$

$$\simeq 32.75\ lit/sec.$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 10 **UNIT IV INCOMPRESSIBLE VISCOUS FLOW**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level
	Water Hammer:	

Teaching Learning Material	Student Activity

Lecture Notes

Water Hammer:-

Water Hammer is a shock wave passing down the pipe as a result of a sudden change in flow rate:

Causes of water Hammer:-

1. Air in pipes
2. Plumbing design faults
3. Wrong types of taps used for particular fittings
4. Restrictions in flow caused by kinks (or) Corrosion in pipes
5. Automatic valves on sprinkler systems
6. Faulty float controls in toilet cisterns
7. Starting and stopping of pumps.

The magnitude of pressure rise due to water hammer depends on the following factors:-

-> The ~~I~~ length of pipe
-> The Speed of valve closure
-> The velocity of flow and
-> The elastic properties of the material of pipe well as the flowing liquid.

while designing the pipe, the pressure rise due to water hammer Should be taken into account.

water hammer in pipes can be eliminated by attaching a valve for the following reasons:

(i) Gradual closure of a valve
(ii) Instantaneous closure of a valve in rigid pipes
(iii) Instantaneous closure of a valve in elastic pipe.

Cavitation:-

phenomenon of Formation of vapour bubbles of a flowing liquid in a Region where the pressure of the liquid falls below its vapour pressure and the Sudden Collapsing of these vapour bubbles occur in a Region of Higher pressure.

The Harmful effects of Cavitation are as follows:-

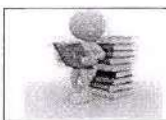
- ✓ Pitting and erosion of Surface are due to Continuous hammering action of the Collapsing bubbles
- ✓ Sudden drop in head, efficiency and power delivered to the fluid occur.
- ✓ Noise and Vibrations are produced by the Collapsing of vapour bubbles.

Precautions against Cavitations:-

- ✓ pressure of the flowing fluid in any part of the hydraulic System Should not be allowed to fall below its Vapour pressure. In the Case of absolute pressure head Should not be allowed to fall. below 2.5m of water.

Special material Coating can be given to the Surface where the Cavitations occurs. The Special materials Such as aluminium - bronze and Stainless Steel are Cavitation resistant materials.

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

Lecture No. 01

UNIT V BOUNDARY LAYERS

	Lecture Outcome (LO)	Bloom's Level
	At the end of this lecture, students will be able to	

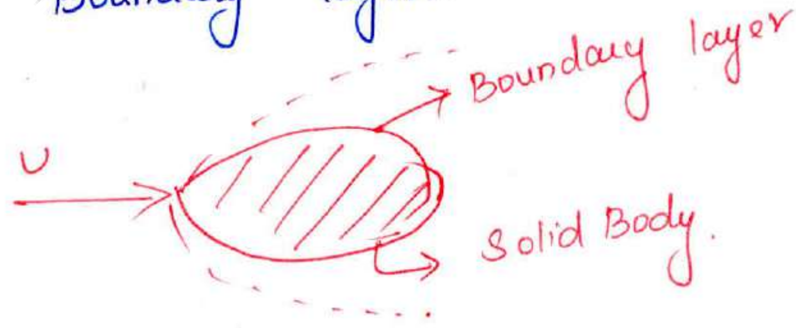
Teaching Learning Material	Student Activity

Lecture Notes

Definition of Boundary layers - Laminar and turbulent Boundary layers - Displacement, momentum and energy thickness - momentum integral equation - Application - Separation of Boundary layer - Drag and Lift forces.

Boundary layers:-

when a Solid body is immersed in a flowing fluid, there is a narrow region of the fluid in the neighbour hood of the Solid body, where a Velocity of fluid Varies from 0 to free Stream Velocity (u). This narrow region of fluid is called Boundary layer:-

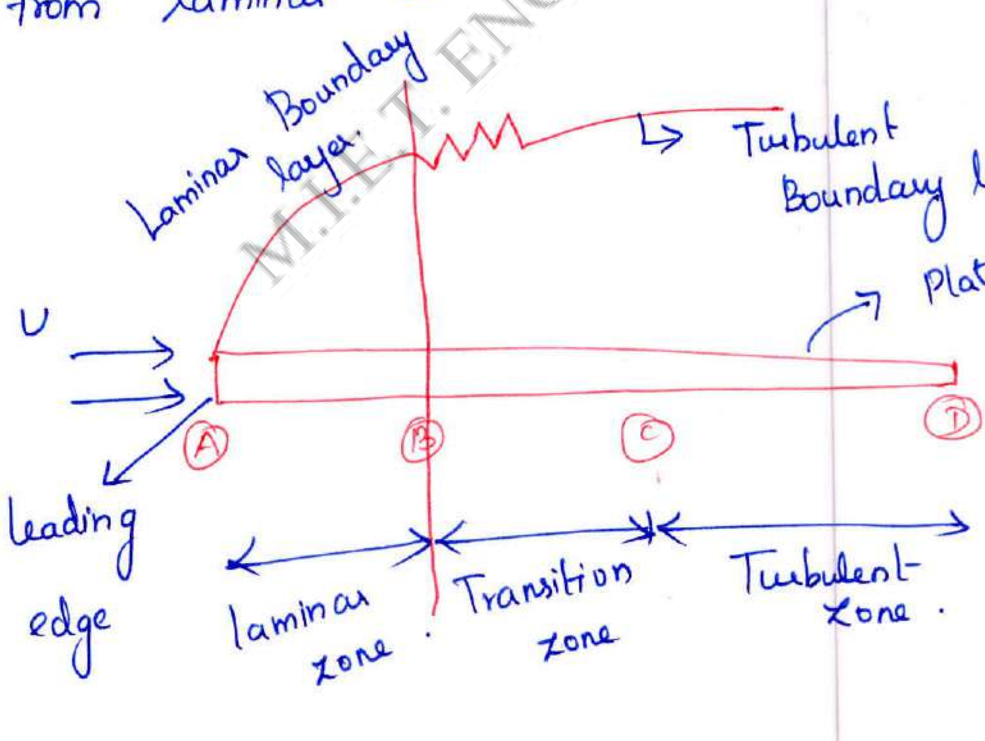


Laminar Boundary layer:-

Near the leading edge of plate, where the thickness is small, the flow in the Boundary layer is laminar. This larger of fluid is set to be laminar Boundary layer.

Turbulent Boundary layer:-

The laminar Boundary layer becomes unstable and motion of fluid gets disturbed and irregular which leads to transistion from irregular. Irregular which leads to transistion from laminar to another which leads to transistion from laminar to another Boundary layer. The layer called as the turbulent Boundary layer. The short length where the Boundary layer changes from laminar to turbulent is called transistion zone.



Boundary layer thickness (δ)

It is defined as the distance from the Boundary of the Solid Body measured in y- direction to the point, where the velocity of fluid is approximately equal to 0.99 times free stream Velocity (U) of the fluid.

It is denoted by the Symbol (δ)

Displacement thickness (δ^*)

It is defined as the distance measured ⊥r to the Boundary of Solid Body, by which the Boundary Should be displaced to Comp. for the reduction in flow rate on account of Boundary layer formation

It is denoted by (δ^*)

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

momentum thickness (θ)

It is defined as the distance measured ⊥r to the Boundary of Solid body, by which the Boundary should be displaced to for the reduction in momentum in flowing fluid on account of Boundary layer formation. It is denoted by θ.

$$\theta = \int_0^{\delta} \frac{u}{V}\left[1 - \frac{u}{V}\right) dy$$

Energy thickness :- (δ^{**})

It is defined as the distance, measured ⊥r to the Boundary of Solid Body by which the Boundary Should be displaced to compansate for the reduction in the kinetic energy of flowing fluid. It is denoted by δ^{**}

$$\delta^{**} = \int_0^{\delta} \frac{u}{V}\left[1 - \frac{u^2}{V^2}\right] dy.$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			
Any other suggested Materials			

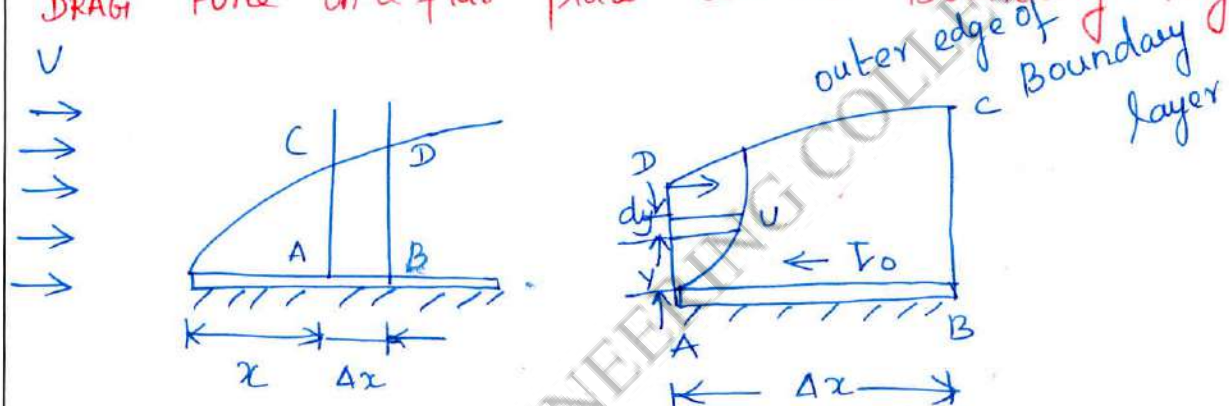
Lecture No. **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

DRAG Force on a flat plate due to Boundary layer:-



$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy\right]$$

In the equation ⇒ $\int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy$ is equal to momentum thickness θ,

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy \Rightarrow \boxed{\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}} \text{ (or) } \frac{\tau_0}{\rho U^2} = \frac{d\theta}{dx}$$

$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$ ⇒ Von Karman momentum Integral equation.

$$\boxed{\frac{\tau_0}{\rho U^2} = \frac{d\theta}{dx}}$$

This equation applied in :-

1. Laminar boundary layers
2. Transition boundary layers and.
3. Turbulent boundary layers. flow

$$\boxed{\Delta F_D = \tau_0 \times \Delta x \times b.}$$

Then total drag on the plate of length L on one side is.

$$F_D = \int \Delta F_D = \int_0^L \tau_0 \times b \times dx \quad (\text{change } \Delta x = dx)$$

Local Co-eff of Drag (C_D^*)

$$\boxed{C_D^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}}$$

Average Co-eff of Drag (C_D)

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}}$$

A = area of the Surface (or) plate
U = Free stream Velocity
ρ = Mass density of fluid

Boundary Conditions for the Velocity Profiles :-

The following are the Boundary Condition which must be satisfied by any velocity profile, whether it is in laminar boundary layer zone, or in turbulent

Boundary layer zone :-

1. At $y = 0$, $u = 0$ and $\frac{du}{dy}$ has Some finite value
2. At $y = \delta$, $u = U$
3. At $y = \delta$, $\frac{du}{dy} = 0$.

, ΔF_D = Shear Stress x Area

3. For the velocity profile for laminar boundary layer flows given as $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ find an expression for Boundary layer thickness (δ) Shear stress (τ_0) and Co-efficient of drag (C_D) in terms of Reynold numbers.

→ (i) The velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \rightarrow ①$

Sub this value of $\frac{u}{U}$ in equation

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right] = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right] dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy\right]$$

$$= \frac{\partial}{\partial x}\int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \frac{\partial}{\partial x}\int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy = \frac{\partial}{\partial x}\left[\frac{2y^2}{2\delta} - \frac{5\times y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \frac{\partial}{\partial x}\left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}\right] = \frac{\partial}{\partial x}\left[\delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{15\delta - 25\delta + 15\delta - 3\delta}{15}\right] = \frac{\partial}{\partial x}\left[\frac{30\delta - 28\delta}{15}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{2\delta}{15}\right] = \frac{2}{15}\left(\frac{\partial}{\partial x}\right)(\delta) \rightarrow \text{II}$$

$$\tau_0 = \rho U^2 \times \frac{2}{15}\frac{\partial}{\partial x}(\delta) = \frac{2}{15}\rho U^2 \frac{\partial[\delta)}{\partial x}$$

The shear stress at the boundary in laminar flow is also given by Newton's law of viscosity as

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} \rightarrow (2)$$

$$u = U\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]$$

$$\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$$

$$\left(\frac{du}{dy}\right)_{y=0} = U\left[\frac{2}{\delta} - \frac{2\times 0}{\delta^2}\right] = \frac{2U}{\delta}$$

Sub

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO)	Bloom's Level
	At the end of this lecture, students will be able to	

Teaching Learning Material	Student Activity

Lecture Notes

Displacement thickness (δ^*)

$$\delta^* = \int_0^{\delta} \left(1-\frac{u}{U}\right) dy$$

momentum thickness:- (θ)

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy$$

Energy thickness:- (δ^{**})

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u^2}{U^2}\right] dy$$

1. The Velocity distribution in the Boundary layer is given by $\frac{u}{U} = \frac{y}{\delta}$ where u = velocity at a distance y from the flat plate and u = U at y = δ where δ = Boundary layer thickness. Determine the Value of: (i) displacement thickness (ii) momentum thickness (iii) energy thickness and (iv) $\frac{\delta^*}{\theta}$

→ (i) displacement thickness δ^*

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy \qquad \left(\because \text{Given } \frac{u}{U} = \frac{y}{\delta}\right)$$

$$= \left[y - \frac{y^2}{2\delta}\right] = \left[\delta - \frac{\delta^2}{2\delta}\right] = \delta - \frac{\delta^{\cancel{2}}}{2\cancel{\delta}}$$

$$= \delta - \frac{\delta}{2}$$

$$= \delta/2$$

(ii) momentum thickness θ :-

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$$

$$= \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y}{\delta}\right] dy = \int_0^{\delta} \left[\frac{y}{\delta} - \frac{y^2}{\delta^2}\right] dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^{\delta} = \left[\frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}\right]$$

$$= \left[\frac{\delta}{2}\right] - \left[\frac{\delta}{3}\right] = \frac{3\delta - 2\delta}{6} = \frac{\delta}{6}$$

(iii) Energy thickness δ_e :-

$$\delta^{**} \text{ (or) } \delta_e = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy$$

$$= \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y^2}{\delta^2}\right] dy = \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) dy$$

$$= \left(\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right)_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3} = \frac{\delta}{2} - \frac{\delta}{4} = \frac{\delta}{4}$$

(iv) $\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = \frac{\cancel{\delta}}{\cancel{2}} \times \frac{\cancel{6}^3}{\cancel{\delta}} = 3$

2. The velocity distribution in laminar boundary layer is given by $\frac{u}{U} = 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$

where u = Velocity at distance y from the boundary

U = Velocity at a distance

δ = Thickness of the boundary layer.

Calculate the:-

(i) ratio of displacement thickness to boundary layer thickness $\left(\frac{\delta^*}{\delta}\right)$

(ii) ratio of momentum thickness to boundary layer thickness $\left(\frac{\theta}{\delta}\right)$.

$\rightarrow$ $\frac{u}{U} = 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$

(i) Ratio of displacement thickness to Boundary layer thickness $\left(\frac{\delta^*}{\delta}\right)$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2\right) dy = \int_0^\delta 1 - 3\left(\frac{y}{\delta}\right) - 2\frac{y^2}{\delta^2}\, dy$$

$$\delta^* = \left[y - 3\left(\frac{y^2}{2\times\delta}\right) - 2\left(\frac{y^3}{3\delta^2}\right)\right]_0^\delta$$

$$= \delta - 3\frac{\delta^2}{2\times\delta} - 2\frac{\delta^3}{3\delta^2} \Rightarrow \delta - \frac{3}{2\delta}\delta^2 - 2\frac{\delta}{3}$$

$$\delta^* = \delta - \frac{3\delta}{2} - \frac{2\delta}{3}$$

$$\delta^* = \frac{6\delta - 9\delta + 4\delta}{6} = \frac{1}{6}(10-9)\delta = \frac{\delta}{6}$$

$$\boxed{\frac{\delta^*}{\delta} = \frac{1}{6}}$$

(ii) Ratio of momentum thickness to Boundary layer thickness (θ/δ)

$$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$$

$$= \int_0^\delta \left[3\frac{y}{\delta} - 2\frac{y^2}{\delta^2}\right]\left[1 - \left(3\frac{y}{\delta} - 2\frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{3y}{\delta} - \frac{2y^2}{\delta^2}\right]\left[1 - \frac{3y}{\delta} + 2\frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{3y}{\delta} - \frac{9y^2}{\delta^2} + \frac{6y^3}{\delta^3} - \frac{2y^2}{\delta^2} + \frac{6y^3}{\delta^3} - \frac{4y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{3y}{\delta} - \frac{11y^2}{\delta^2} + \frac{12y^3}{\delta^3} - \frac{4y^4}{\delta^4}\right] dy$$

$$= \left[\frac{3y^2}{2\delta} - \frac{11y^3}{3\delta^2} + \frac{12y^4}{4\delta^3} - \frac{4y^5}{5\delta^4}\right]_0^\delta$$

$$= \frac{3\delta^2}{2\delta} - \frac{11\delta^3}{3\delta^2} + \frac{12\delta^4}{4\delta^3} - \frac{4\delta^5}{5\delta^4}$$

$$= \frac{3}{2}\delta - \frac{11}{3}\delta + 3\delta - \frac{4}{5}\delta$$

$$= \frac{(15\times3)\delta - 110\delta + 90\delta - 24\delta}{30}$$

$$= \frac{45 - 110 + 90 - 24}{30}(\delta) = \frac{135 - 134}{30} = \frac{1}{30}$$

$$\boxed{\theta/\delta = 1/30}$$

Suggested Questions / Assignments / Home works / any other

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

UNIT V BOUNDARY LAYERS

Lecture No.	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

The velocity distribution in a laminar Boundary layer is given by $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$. Calculate the displacement thickness, momentum thickness and energy thickness.

→ velocity distribution $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$

(i) displacement thickness, δ^*

w.k.that.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \int_0^\delta 1 - \frac{3y}{\delta} + \frac{y^2}{2\delta^2}\, dy = \left[y - \frac{3y^2}{4\delta} + \frac{y^3}{6\delta^2}\right]_0^\delta$$

$$= \delta - \frac{3\delta^{\cancel{2}}}{4\delta} + \frac{\delta^3}{6\delta^2} = \delta - \frac{3\delta}{4} + \frac{\delta}{6}$$

$$= \frac{12 - 18\delta +}{6} \quad \frac{24\delta - 18\delta + 4\delta}{24} = \frac{(28-18)\,\delta}{24}$$

$$= \frac{10}{24}\delta = \frac{5}{12}\delta$$

$$\delta^* = \frac{5}{12}\delta \text{ (or) } \frac{10}{24}\delta$$

b) momentum thickness θ

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]\left(1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right) dy$$

$$= \int_0^\delta \left[\frac{3y}{2\delta} - \frac{y^2}{2\delta^2}\right]\left[1 - \frac{3y}{2\delta} + \frac{y^2}{2\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2} + \frac{3y^3}{4\delta^3} - \frac{y^2}{2\delta^2} + \frac{3y^3}{4\delta^3} - \frac{y^4}{4\delta^4}\right] dy$$

$$= \frac{3}{2\delta} \times \frac{y^2}{2} - \frac{9}{4\delta^2} \times \frac{y^3}{3} + \frac{3}{4\delta^3} \times \frac{y^4}{4} - \frac{1}{2\delta^2} \times \frac{y^3}{3} + \frac{3}{4\delta^3} \times \frac{y^4}{4} - \frac{1}{4\delta^4} \times \frac{y^5}{5}$$

$$= \frac{3}{2\delta} \times \frac{\delta^2}{2} - \frac{9}{4\delta^2} \times \frac{\delta^3}{3} + \frac{3}{4\delta^3} \times \frac{\delta^4}{4} - \frac{1}{2\delta^2} \times \frac{\delta^3}{3} + \frac{3}{4\delta^3} \times \frac{\delta^4}{4} - \frac{1}{4\delta^4} \times \frac{\delta^5}{5}$$

$$= \frac{3\delta}{4} - \frac{9\delta}{12} + \frac{3\delta}{16} + \frac{2\delta}{6} + \frac{3\delta}{16} - \frac{\delta}{20}$$

$$= \frac{180\delta - 180\delta + 45\delta - 40\delta + 45\delta - 12\delta}{240}$$

$$= \frac{38\delta}{240}$$

$$\theta = \frac{19}{120}\delta$$

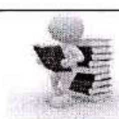
C Energy thickness (δ_e)

$$\delta_e = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u^2}{U^2}\right] dy .$$

$$= \int_0^{\delta} \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \left[1- \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right)^2 dy .$$

$\frac{63\delta}{240}$.

Suggested Questions / Assignments / Home works / any other



Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

The velocity distribution in the Boundary layer is given by $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/m}$ calculate the (i) displacement thickness and (ii) momentum thickness in terms of the Boundary layers thickness δ.

→ Velocity distribution $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/m}$

(i) Displacement thickness (δ^*)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/m}\right) dy$$

$$= \left[y - \frac{1}{\delta^{1/m}}\left[\frac{m}{m+1}\right] y^{\frac{m+1}{m}}\right]_0^\delta$$

$$= \left[\delta - \frac{1}{\delta^{1/m}} \times \frac{m}{m+1}\, \delta^{\frac{m+1}{m}}\right]$$

$$= \left[\delta - \left[\delta^{-1/m} \times \frac{m}{m+1}\, \delta^{\frac{m+1}{m}}\right]\right] = \delta - \frac{m}{m+1}\, \delta^{-1/m}\, \delta^{\frac{m+1}{m}}$$

$$= \delta - \frac{m}{m+1}\, \delta^{\frac{-1+m+1}{m}} = \left[\delta - \frac{m}{m+1}\, \delta\right] = \delta\left(1 - \frac{m}{m+1}\right)$$

$$= \delta\left(\frac{\cancel{m} + 1 - \cancel{m}}{m+1}\right) = \delta\left(\frac{1}{m+1}\right)$$

$$x^n dx = \frac{x^{n+1}}{n+1}$$

$$(y)^{1/m} = \frac{y^{\frac{1}{m}+1}}{\frac{1}{m}+1} = \frac{y^{\frac{1+m}{m}}}{\frac{1+m}{m}} \Leftarrow \frac{m}{1+m}\, y^{\frac{m+1}{m}}$$

(ii) momentum thickness (θ):-

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/m}\left[1-\left(\frac{y}{\delta}\right)^{1/m}\right] dy$$

$$= \int_0^{y} \left[\left(\frac{y}{\delta}\right)^{1/m} - \left(\frac{y}{\delta}\right)^{2/m}\right] dy$$

$$= \left[\frac{1}{\delta^{1/m}} \times \frac{m}{m+1}\, y^{\frac{m+1}{m}} - \frac{1}{\delta^{2/m}} \times \frac{m}{m+2}\, y^{\frac{m+2}{m}}\right]_0^{\delta}$$

$$= \left[\frac{1}{\delta^{1/m}} \times \frac{m}{m+1}\, \delta^{\frac{m+1}{m}} - \frac{1}{\delta^{2/m}} \times \frac{m}{m+2}\, \delta^{\frac{m+2}{m}}\right]$$

$$= \left[\frac{1}{\delta^{1/m}} \times \frac{m}{m+1}\, \delta^{\frac{m+1}{m}} - \frac{1}{\delta^{2/m}} \times \frac{m}{m+2}\, \delta^{\frac{m+2}{m}}\right]$$

$$= \frac{m}{m+1}\, \delta^{-1/m}\, \delta^{\frac{m+1}{m}} - \frac{m}{m+2}\, \delta^{-2/m}\, \delta^{\frac{m+2}{m}}$$

$$= \frac{m}{m+1}\left[\delta^{\frac{\cancel{m}+\cancel{1}-\cancel{1}}{\cancel{m}}}\right] - \frac{m}{m+2}\left[\delta^{\frac{-\cancel{2}+\cancel{m}+\cancel{2}}{\cancel{m}}}\right]$$

$$= \frac{m}{m+1}\,\delta - \frac{m}{m+2}\,\delta$$

$$= \delta\left[\frac{m}{m+1} - \frac{m}{m+2}\right] = \left[\frac{m(m+2) - m(m+1)}{(m+1)(m+2)}\right]$$

$$= \delta\left[\frac{\cancel{m^2}+2m-\cancel{m^2}-m}{(m+1)(m+2)}\right] = \left[\frac{m}{(m+1)(m+2)}\right]\delta$$

The Velocity distribution of the Boundary layer over Spillway is observed as $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{0.22}$. The main stream velocity U at a Certain Section was observed to be 25 m/s and the boundary layer thickness of 50mm was estimated from the velocity distribution measured at the Section. The discharge passing over the Spillway was $5\,m^3/s$ per metre length of the Spillway. Calculate the displacement thickness, momentum thickness, energy thickness and loss of energy upto the Section under Consideration:-

Given :-

Velocity distribution $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{0.22}$

mainstream velocity $\Rightarrow U = 25\,m/s$

Boundary layer thickness $-\delta = 50mm$

Discharge $q = 5\,m^3/s$

$\rightarrow$

(i) Displacement thickness (δ^*)

$$\delta^* = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy \Rightarrow \int_0^{\delta}\left[1-\left(\frac{y}{\delta}\right)^{0.22}\right]dy$$

$$= \left[y - \left[\frac{1}{\delta^{0.22}}\right] \times \frac{y^{1.22}}{1+0.22}\right]_0^{\delta}$$

$$= \delta - \left(\frac{1}{\delta^{0.22}} \times \frac{\delta^{1.22}}{1.22}\right]$$

$$= \delta - \delta^{-0.22} \times \frac{\delta^{1.22}}{1.22}$$

$$= \delta - \frac{\delta^{-0.22} \times \delta^{1.22}}{1.22}$$

$$= \delta - \left(\frac{1}{1.22}\right)\left(\delta^{1.22-0.22}\right)$$

$$= \delta - \frac{1}{1.22}\left(\delta^1\right)$$

$$= \delta - \frac{\delta}{1.22} = \delta\left(1-\frac{1}{1.22}\right)$$

$$= \delta\left[\frac{1.22-1}{1.22}\right] = 0.18\delta$$

$(\because \delta = 50mm)$

$$\boxed{\delta^* = 0.18 \times 50 = 9mm.}$$

(ii) momentum thickness.

$$\theta = \int_0^{\delta}\frac{u}{U}\left(1-\frac{u}{U}\right)dy.$$

$$= \int_0^{\delta}\left(\frac{y}{\delta}\right)^{0.22}\left[1-\left(\frac{y}{\delta}\right)^{0.22}\right]dy$$

$$= \int_0^{\delta}\left(\frac{y}{\delta}\right)^{0.22} - \left(\frac{y^{0.44}}{\delta^{0.44}}\right)dy$$

$$= \left[\frac{1}{\delta^{0.22}}\left(\frac{y^{1.22}}{1.22}\right) - \frac{1}{\delta^{0.44}}\left(\frac{y^{1.44}}{1.44}\right)\right]_0^{\delta}$$

$$= \left[\frac{\delta^{1.22}}{1.22\times\delta^{0.22}} - \frac{\delta^{1.44}}{1.44\times\delta^{0.44}}\right]$$

$$= \left(\frac{\delta^{1.22}\,\delta^{-0.22}}{1.22}\right) - \left(\frac{\delta^{1.44}\,\delta^{-0.44}}{1.44}\right)$$

$$= \frac{\delta^{1}}{1.22} - \frac{\delta}{1.44} = \delta\left(\frac{1}{1.22} - \frac{1}{1.44}\right)$$

$$= 0.125\,\delta = 0.125\times 50 = 6.25\ \text{mm}.$$

(iii) Energy thickness (δ_e):-

$$\delta_e = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy$$

$$= \int_0^{\delta}\left(\frac{y}{\delta}\right)^{0.22}\left[1 - \left(\frac{y}{\delta}\right)^{0.44}\right] dy$$

$$= \int_0^{\delta}\left[\left(\frac{y}{\delta}\right)^{0.22} - \left(\frac{y}{\delta}\right)^{0.66}\right] dy$$

$$\left[\frac{1}{\delta^{0.22}}\left(\frac{y^{1.22}}{1.22}\right) - \frac{1}{\delta^{0.66}}\left(\frac{y^{1.66}}{1.66}\right)\right]_0^{\delta}$$

$$= \frac{1}{\delta^{0.22}}\left[\frac{\delta^{1.22}}{1.22}\right] - \frac{1}{\delta^{0.66}}\left[\frac{\delta^{1.66}}{1.66}\right]$$

$$= \frac{\delta^{-0.22}\,\delta^{1.22}}{1.22} - \frac{\delta^{-0.66}\,\delta^{1.66}}{1.66}$$

$$= \frac{\delta}{1.22} - \frac{\delta}{1.66} = \delta\left[\frac{1}{1.22} - \frac{1}{1.66}\right]$$

$$= 0.217\,\delta = 0.217\times 50 = 10.85\ \text{mm}$$

(iv) Energy loss per metre of spillway.

$$E_L = \frac{1}{2}(\rho\,\delta_e\,U)\times U^2 = \frac{1}{2}\rho\,\delta_e\,U^3$$

$$E_L = \frac{1}{2}\rho\,\delta_e\,U^3 = \frac{1}{2}\times 1000\times 10.85\times 25^3$$

$$= 84765.63\ \text{W} \Rightarrow \underline{84.765\ \text{kW}}$$

(v) Energy loss in terms of metre of head:

$$\frac{E_L}{Wq} = \frac{84765.63}{5\times 9810} = 1.73\ \text{m}$$

$W = 9810$ $\quad W = \rho\times g$

$W = 1000\times 9.81$

$= 9810$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 05 **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

A plate of length 750mm and width 250mm has been placed longitudinally in a stream of crude oil which flows with a velocity of 5m/s. If the oil has a Sp. gravity of 0.8 & kinematic viscosity of 1 stoke, calculate the

(i) boundary layer thickness at the middle of the plate

(ii) Shear stress at the middle of the plate

(iii) Friction drag on one side of the plate:-

Given:-

Length of the plate, $L = 750mm = 0.75m$

width of the plate $b = 250mm = 0.25m$

Velocity, $U = 5\ m/s$

Specific gravity, $S = 0.8$

Kinematic Viscosity, $\nu = 1$

Stoke $= 1 \times 10^{-4} m^2/s$

Solution:-

(i) Boundary layer thickness at the middle of the plate :-

at the middle of the plate

$$x = \frac{0.75}{2} = 0.375\,m.$$

$$\boxed{Re = \frac{Ux}{\nu}}$$

$$Re = \frac{5 \times 0.375}{0.0001} = 18750 < 5\times10^5$$

Note:

Turbulent Boundary layer:-

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^n$$ where $n = \frac{1}{7}$ approximately for $Re < 10^7$ $\cancel{>}$ 5×10^5

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

is known as one - Seventh power law.

$$\tau_0 = 0.0226\,\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$$ For $Re = 5\times10^5$ to $\times 10^7$

① $\boxed{Re = \frac{Ux}{\nu}}$ U = Velocity $x_1 = L/2$ (middle of the plate)

ν = kinematic viscosity.

② Laminar Boundary layer thickness

$$\boxed{\delta_{lam} = \frac{5x}{\sqrt{Re}}}$$

(ii) Shear stress at the middle of the plate

By ; Blasius theory, the local co-efficient of drag.

is given by.

$$\boxed{C_D^* = \frac{0.664}{\sqrt{Re_x}}}$$

$$\boxed{\tau_0 = C_D^* \frac{1}{2} \rho U^2}$$

(iii) Friction drag on one side of the plate:-

At the trailing edge of the plate L = length of the plate.

$$\boxed{Re_L = \frac{UL}{\nu}}$$

✓ average drag co-efficient $C_D^* = \frac{1.328}{\sqrt{Re_L}}$

✓ Friction drag force = Stress x Area

$$F_D = \int_0^L \tau_0 \times b \times dx$$

$$F_D = C_D^* \times \frac{1}{\rho} U^2 \times b \times L$$

→

So, the Boundary layer is laminar Boundary layer.

$\boxed{\delta_{Lam} = \frac{5x}{\sqrt{Re}}} = \frac{5 \times 0.375}{\sqrt{18750}} = 0.0137m = 13.7mm$

(ii) Shear stress @ the middle

Blasius theory;

$\boxed{C_D^* = \frac{0.664}{\sqrt{Re_x}}} = \frac{0.664}{\sqrt{18750}} = 4.85 \times 10^{-3}$

we know that

sp. gravity of oil $\boxed{S = \frac{\rho_{oil}}{\rho_{water}}}$

$0.8 = \frac{\rho_{oil}}{1000} = 800 kg/m^3.$

shear stress

$\boxed{\tau_0 = C_D^* \frac{1}{2} \rho U^2}$

$= 4.85 \times 10^{-3} \times \frac{1}{2} \times 800 \times 5^2$

$= 4.85 N/m^2$

(iii) Friction drag on one side of the plate

At the trailing edge of the plate $L = 0.75m$

$\boxed{Re_L = \frac{UL}{\nu}} = \frac{5 \times 0.75}{0.0001}$

$= 37500 < 5 \times 10^5$

so the boundary layer is also the laminar boundary layer at the trailing edge.

average drag co-eff

$C_D^* = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{37500}} = 6.858 \times 10^{-3}$

∴ Friction drag force = stress × Area

$F_D = \int_0^L \tau_0 \times b \times dx$

$F_D = C_D^* \times \frac{1}{2} \rho U^2 \times b \times L$

$F_D = 6.858 \times 10^{-3} \times \frac{1}{2} \times 800 \times 5^2 \times 0.25 \times 0.75$

$\boxed{F_D = 12.86 N}$

Suggested Questions / Assignments / Home works / any other

A plate of 600mm length and 400mm wide is immersed in a fluid of sp. gravity 0.9 and kinematic viscosity of $\nu = 10^{-4} m^2/s$. The fluid is moving with the velocity of 6m/s. determine the:- (i) boundary layer thickness (ii) shear stress at the end of the plate & (iii) drag force on one of the side of the plate

Text Books / Reference Books

S.No	Title	Author	Publisher
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

Any other suggested Materials

Lecture No. 7 **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

$\not{x}$. mech Dec 07 & Dec 14, civil Nov 14, may 17.

A flat plate 1·5m × 1·5m moves at 50 km/h in a stationary air of density 1·15 kg/m³. If the co-eff of drag & lift are 0·15 and 0·75 respectively, determine (i) the life force (ii) the drag force (iii) the Resultant force and (iv) the power required to set the plate in motion.

Given data:-

Area, $a = 1.5 \times 1.5 = 2.25 m^2$

Velocity, $V = 50\ km/hr = \frac{50 \times 1000}{3600} = 13.89\ m/s$

Co-eff of drag $C_D = 0.15$

Co-eff of lift $C_L = 0.75$

Density of air $\rho = 1.15 kg/m^3$

→ Solution:

Sp. wt, $w = \rho \times g = 1.15 \times 9.81 = 11.282 N/m^3$

Drag Force $\boxed{F_D = C_D \times \frac{w a v^2}{2g}}$

$$= 0.15 \times \frac{11.282 \times 2.25 \times 13.89^2}{2 \times 9.81}$$

$$= 37.44 N$$

Lift Force: $\boxed{F_L = C_L \times \frac{w a v^2}{2g}}$

$$= 0.75 \times \frac{11.282 \times 2.25 \times 13.89^2}{2 \times 9.81} = 187.21 N$$

$\boxed{\text{Resultant Force} = F_R = \sqrt{F_L^2 + F_D^2}}$

$$= \sqrt{(37.44)^2 + (187.21)^2} = 190.92 N.$$

Power Required $= \boxed{P = F_D \times U}$

$P = 37.44 \times 13.89 = 520.04$ W (or) 0.52 kN

Civil may 2016.

3. A flat plate of 2m width and 4m length is kept parallel to air flowing at 5 m/s Velocity at 15°c. Determine the length of a plate over which Boundary layer is Laminar, Shear at the location where the Boundary layer Ceases to be laminar & total force on both Sides on that portion of the plate where the Boundary layer is laminar. Take $\rho = 1.208 \ kg/m^3$ & $\nu = 1.47 \times 10^{-5} m^2/s$.

→ Velocity, $U = 5 \ m/s$

length $L = 4m$

width $b = 2m$

kinematic Viscosity $\nu = 1.47 \times 10^{-5} m^2/s$

density $\rho_{air} = 1.208 \ kg/m^3$

Lecture No. **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Solution:

Reynold's number, $\boxed{Re = \frac{UL}{\nu}} = \frac{5\times4}{1.47\times10^{-5}} = 1360544.22$

Since $Re > 5\times10^5$, the flow is turbulent But the flow entry is laminar, So, the thickness of the laminar flow is found by

$$5\times10^5 = \frac{5x}{1.47\times10^{-5}} \Rightarrow x = 1.47m$$

Boundary layer thickness, $\boxed{\delta_{lam} = \frac{5x}{\sqrt{Re}}} = \frac{5\times1.47}{\sqrt{5\times10^5}}$

$= 0.0104m = 10.4mm$

Blasius theory, the local Co-efficient of drag is given by $\boxed{C_D^* = \frac{0.664}{\sqrt{Re}}} = \frac{0.664}{\sqrt{5\times10^5}} = 9.39\times10^{-4}$

$$\boxed{\text{Shear Stress}, \tau_o = C_D^* \frac{1}{2}\rho U^2}$$

$= 9.39\times10^{-4}\times\frac{1}{2}\times1.208\times5^2 = 0.0142\ N/m^2$

Friction drag Force, F_D = Stress x Area. Force acting on both Sides.

$F_D = 2\int_0^4 \tau_o \times b\times dx$.

$= 2\int_0^4 \tau_o\times b\times L = 2\times0.0142\times2\times4$

$= 0.23N$.

Civil may 2016

A Smoth rectangular plate of 1m width and 20m length when towed through water at 20°c lengthwise experience along drag of 1440 N on both the sides. Determine the (i) average drag Co-eff (ii) velocity of the plate (iii) Boundary layer thickness at the edage of the plate.

Given data:

width, $b = 1m$

Length $L = 20m$

Drag force $F_D = 1440N$

Solution:-

Drag Force

$F_D = \text{stress} \times \text{Area}$

$F_D = 2\int_0^L \tau_0 \times b \times dx$

[∵ Force is acting on both sides)

$= \cancel{2} C_D \times \frac{1}{\cancel{2}} \rho U^2 \times b \times L$

$= C_D \times \rho U^2 \times b \times L$

$1440 = C_D \times 1000\, U^2 \times 1 \times 20$

$C_D U^2 = 0.072$

$\boxed{C_D U^2 = 0.072} \rightarrow ①$

The value of C_D is Not given so assumed Suitably

$\therefore C_D = 0.0015$

$U^2 = \frac{0.072}{0.0015}$

$U = 6.93 m/s$.

For water:-

$\nu = 1 \times 10^{-6}\, m^2/s$

Reyolds number,

$Re = \frac{UL}{\nu} = \frac{6.93 \times 20}{1 \times 10^{-6}} = 138600000$

For High value of Re, C_D can be computed

By. $C_D = \frac{0.455}{(\log Re)^{2.58}}$

$= \frac{0.455}{(\log 138600000)^{2.58}}$

$C_D = 0.00203$

apply in equation ①

$U^2 = \frac{0.072}{0.00203}$

$U = 5.96 m/s$.

$||^{ly}$ $Re = \frac{UL}{\nu} = \frac{5.96 \times 20}{1 \times 10^{-6}} = 119200000$

$\delta_{tur} = \frac{0.22L}{(Re)^{1/6}}$

$= 185.31 mm$

Lecture No. **UNIT V BOUNDARY LAYERS**

	Lecture Outcome (LO) At the end of this lecture, students will be able to	Bloom's Level

Teaching Learning Material	Student Activity

Lecture Notes

Air is flowing over a flat plate with a velocity of 5m/s. The length of the plate is 1.5m and width 1m. The kinematic viscosity of air is given as $0.158\times10^{-4}\ m^2/s$ find the

(i) The boundary layer thickness at the end of plate

(ii) Shear stress at 20cm from the leading edge and

(iii) drag force on one side of the plate.

Take the velocity profile over a plate as $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$ and density of air $1.24\ kg/m^3$.

→ Given data:

Velocity $U = 5\ m/s$

Length $\Rightarrow L = 1.5\ m$

width $b = 1m$

kinematic Viscosity $\nu = 0.158\times10^{-4}\ m^2/s$

Velocity profile $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

$\rho_{air} = 1.24\ kg/m^3$

Solution:

Reynolds number:-

$$R_e = \frac{UL}{\nu} = \frac{5\times1.5}{0.158\times10^{-4}}$$

$$= 4.74\times10^{5}$$

Since $Re < 5\times10^5$, the flow is laminar ∴ the thickness of Boundary layer and shear stress for laminar flow for given velocity profile is obtained as follows.

$$\boxed{\frac{\tau_o}{\rho U^2} = \frac{d}{dx}\left[\int_0^{\delta}\frac{u}{U}\left(1-\frac{u}{U}\right)dy\right]} \rightarrow (1)$$

Subsituting the value of $\frac{u}{U}$ in equation.

$$\frac{\tau_o}{\rho U^2} = \frac{d}{dx}\left[\int_0^{\delta}\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left(1-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right)dy\right]$$

$$= \frac{d}{dx}\left[\int_0^{\delta}\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]dy$$

$$= \frac{d}{dx}\left[\int_0^{\delta}\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\frac{1-\cos2\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{2}\right]$$

$$\boxed{\because \sin^2\theta = \frac{1-\cos2\theta}{2}}$$

$$= \frac{d}{dx}\left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{}\right]$$

$$\int \sin 2x\,dx \quad \sin^2 x = \int\frac{1}{2}\,1-\cos2x$$

$$= \frac{1}{2}\int 1-\cos2x\,dx.$$

$$= \frac{1}{2}\left[x - \cancel{\cos}\,\frac{\sin2x}{2}\right]$$

$$= \frac{1}{2}x - \frac{1}{2}\times\frac{\sin2x}{2}.$$

$$= \frac{1}{2}x - \frac{\sin2x}{4}.$$

diff.

$$= \frac{d}{dx}\left[\int_0^{\delta}\left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\frac{\left(1-\cos\cancel{2}\left(\frac{\pi}{\cancel{2}}\frac{y}{\delta}\right)\right)}{2}\right]dy\right]$$

$$= \frac{d}{dx}\left[\int_0^{\delta}\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\frac{1}{2}\left(1-\cos\frac{\pi y}{\delta}\right)dy.\right]$$

$$= \frac{d}{dx}\left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} - \int_0^{\delta}\left[\frac{1}{2}-\frac{1}{2}\cos\frac{\pi y}{\delta}\right]dy\right]$$

$$= \frac{d}{dx}\left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} - \frac{y}{2}+\frac{1}{2}\,\frac{\sin\frac{\pi y}{\delta}}{\frac{\pi}{\delta}}\right]$$

$$= \frac{d}{dx}\left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} - \frac{y}{2}+\frac{1}{2}\,\frac{\sin\frac{\pi y}{\delta}}{\frac{\pi}{\delta}}\right]_0^{\delta}.$$

$$= dx\left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} + \left(\frac{\cos\frac{\pi}{2\delta}\times 0}{\frac{\pi}{2\delta}}\right)\right\}$$

$$-\frac{\delta}{2} + \frac{1}{2}\left\{\frac{\sin\left(\frac{\pi\delta}{\delta}\right)}{\frac{\pi}{\delta}} - \frac{\sin\left(\frac{\pi}{\delta}\times 0\right)}{\frac{\pi}{\delta}}\right]$$

$$= \frac{d}{dx}\left\{\left(0 + \frac{1}{\frac{\pi}{2\delta}}\right) - \frac{\delta}{2} + (0+0)\right)$$

$$= \frac{d}{dx}\left(\frac{2\delta}{\pi} - \frac{\delta}{2}\right)$$

$$= \left(\frac{4-\pi}{2\pi}\right)\frac{d\delta}{dx}$$

$$\tau_0 = \rho U^2\left(\frac{4-\pi}{2\pi}\right)\frac{d\delta}{dx} \rightarrow (2)$$

w.k. that

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$u = U\left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]$ from velocity distribution

differentiating the above equation with respect to 'y'

$$\frac{du}{dy} = U \times \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \times \frac{\pi}{2\delta}$$

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$u = U\left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]$ from velocity distribution.

differentiating the above equation with respect to 'y'

$$\frac{du}{dy} = U \times \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \times \frac{\pi}{2\delta}$$

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$= \mu U \times \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \times \frac{\pi}{2\delta}$$

$$\tau_0 = \frac{\mu U \pi}{2\delta}$$

To find boundary layer thickness the shear stress in equation (1) and (2).

$$\rho U^2\left(\frac{4-\pi}{2\pi}\right)\frac{d\delta}{dx} = \frac{\mu U\pi}{2\delta}$$

$$\delta \cdot d\delta = \left(\frac{2\pi}{4-\pi}\right)\frac{\mu U\pi}{2\rho U^2}\,dx$$

$$\delta\, dx = \left(\frac{2\pi^2}{4-\pi}\right)\frac{0.158\times10^{-4}}{2\times1.24\times5}\,dx$$

$$\delta \cdot d\delta = 2.93\times10^{-5}\,dx.$$

$$\frac{\delta^2}{2} = 2.93\times10^{-5}x + C.$$

when $x = 0,\ \delta = 0 \quad C \Rightarrow 0.$

So, the equation reduce to

$$\frac{\delta^2}{2} = 2.93\times10^{-5}x$$

$$\delta = \sqrt{2\times2.93\times10^{-5}x}$$

$$\delta = 7.66\times10^{-3}\sqrt{x}$$

(i) The boundary layer thickness at the end of plate $x = L = 1.5$ m.

$$\delta = 7.66\times10^{-3}\sqrt{1.5}$$

$$\delta = 0.0094\,m.$$

Suggested Questions / Assignments / Home works / any other

	Text Books / Reference Books		
S.No	**Title**	**Author**	**Publisher**
1.	Hydraulics and Fluid Mechanics	Modi P.N and Seth	Tata McGraw Hill
2.			
3.			

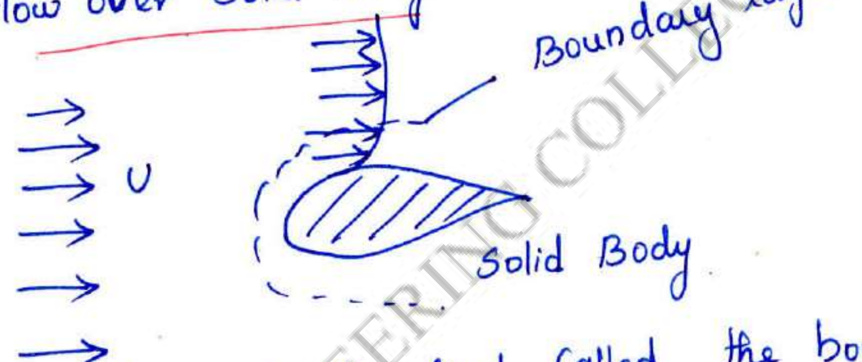
Any other suggested Materials

Boundary layer :-

Definition of Boundary Layers - Laminar and turbulent boundary layers - Displacement, momentum and enery thickness - momentum integral equation - Application Separation of boundary layer - Drag and lift Force.

Definition of Boundary layers.

Flow over Solid Body.



1. A very thin layer of the fluid called the boundary layer in the immediate neighbourhood of the Solid boundary.

✓ where the variation of velocity from 0 to the solid boundary to free - stream velocity in the direction normal to the boundary takes place.

✓ $\boxed{\text{Velocity gradient } \frac{du}{dy}}$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

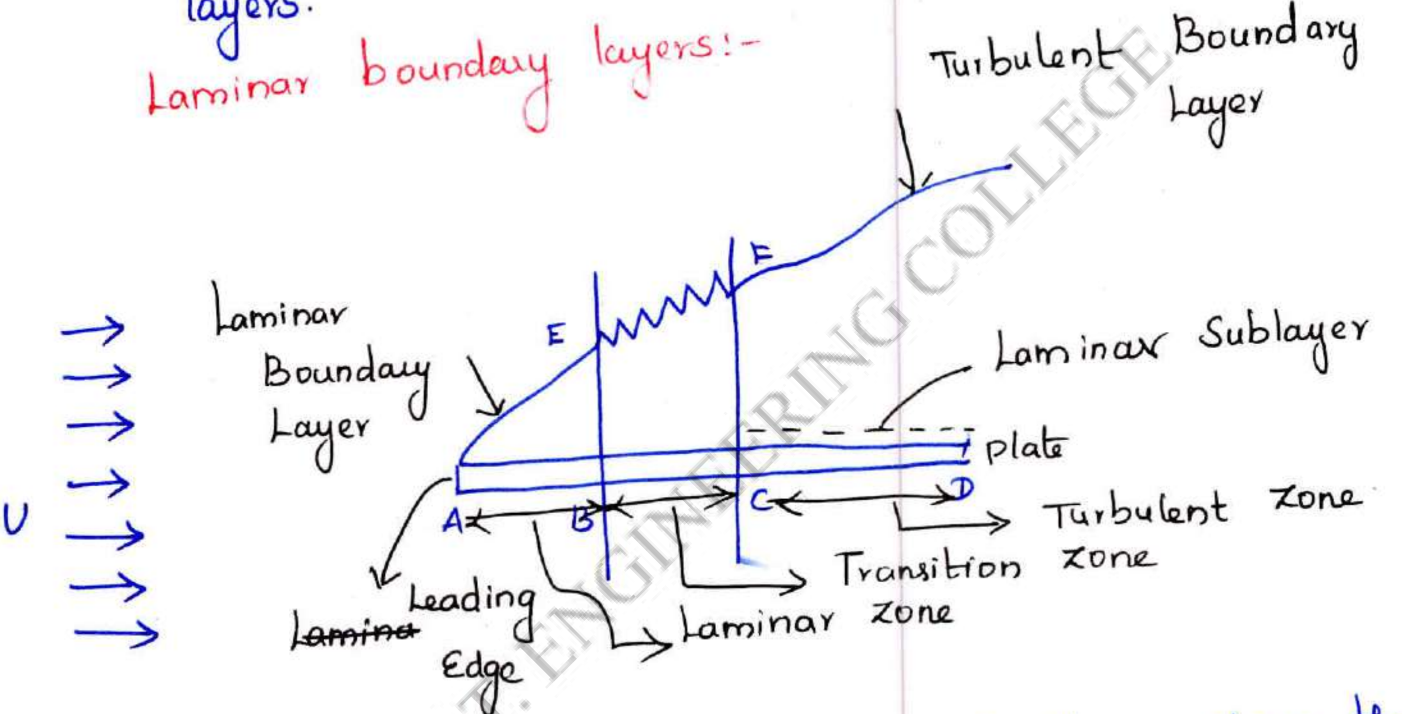
2. The Remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free - stream velocity.

The Result of this the Shear Stress is zero.

Boundary layers:-

when a Solid Body is immersed in a flowing Fluid, there is a narrow ~~resig~~ region of the fluid in the neighbour hood of the Solid body, where a velocity of fluid Varies from 0 to free stream velocity (u) is called boundary. This narrow region of fluid layers.

Laminar boundary layers:-



Near the leading edge of plate, where the thickness is Small, the flow in the Boundary layer is laminar. This layer of Fluid is set to be laminar boundary layer.

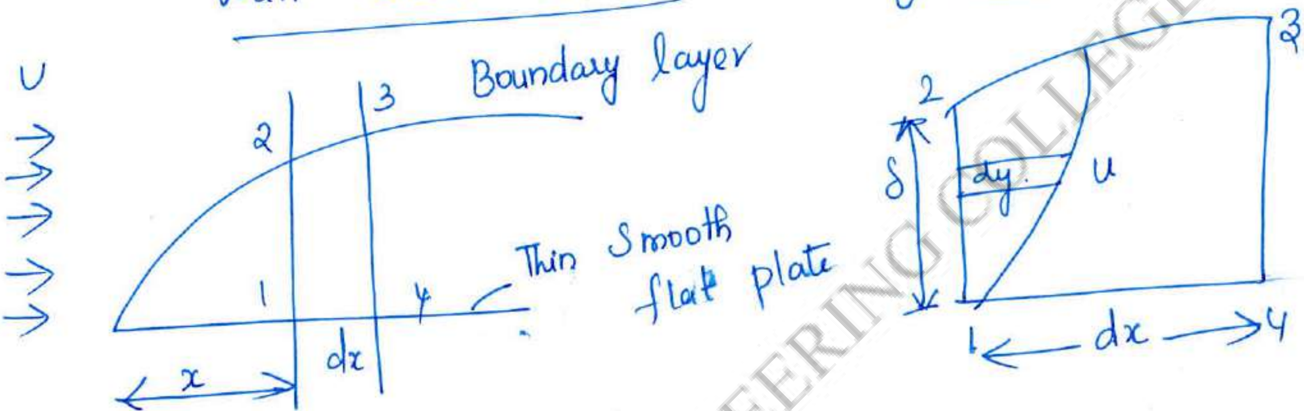
Von-Karman Momentum Integral Equation (or) Drag Force on A Flat plate due to Boundary Layer.

Von - karman $\xrightarrow{\text{Suggested}}$ New method on the basis of momentum equation By using.

growth of the boundary layer

⇓

wall shear stress and drag force are determined.



drag Force

$$\Delta F_D = \tau_o \times dx$$

$$\frac{\tau_o}{\rho U^2} = \frac{d}{dx}\left[\int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy\right]$$

$$\int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right] dy = \text{momentum thickness} (\theta)$$

$$\boxed{\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}}$$ Von - karman momentum Integral equation.

Is used to find out the frictional drag on a Smooth flat plate for both laminar layer and turbulent Boundary layer.

Turbulent Boundary layers:-

The laminar Boundary layer becomes unsatable and motion of fluid gets disturbed and irregular which leads to transistion from laminar to another layer called as the turbulent boundary layer. The Short length where the boundary layers changes from laminar to turbulent is called transistion zone.

Laminar - Sub layer:-

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\frac{u}{y}$$

linear variation $\left\{\frac{\partial u}{\partial y} = \frac{u}{y}\right\}$

Boundary Layer Thickness (δ)

Turbulent Boundary Layer:-

(i) Turbulent Boundary layers are thicker when Compared to Laminar Boundary layers

(ii) Velocity distribution is more uniform. than the L.B. Layers

(iii) Followed, logarithmic law

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^n$$

where $n = \frac{1}{7}$ approximately for $Re < 10^7 > 5\times10^5$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

It is known as one - seveth power law.

The Boundary Conditions must be Satisified for any Velocity distribution as follow:-

(i) At the Surface of the plate

$$y=0,\ u=0 \text{ and } \frac{du}{dy} = \text{Finite Value}$$

ii) At the outer edge of Boundary layer:-

$$y=\delta,\ u=U \text{ and } y=\delta,\ \frac{du}{dy}=0$$

Now a drag force on a Small distance dx of a plate is given by

$$\Delta F_D = \text{Shear Stress} \times \text{Area}$$

$$F_D = \int \Delta F_D = \int_0^L \tau_0 \times B \times dx$$

Local Co-eff of drag:-

$$C_D^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}$$

Average Co-eff of drag:-

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

ρ = mass density of fluid

A = Area of Surface

U = Free Stream Velocity.

VQ: 2022

2.

Seperation of Boundary layers:-

- when a Solid body is immersed in a flowing fluid, a thin layer of fluid called Boundary layer is formed adjacent to Solid Body.
- At Centre pt, a Stage may come where the Boundary layer will be Separated from line Surface due to decrease in velocity. This phenomenon is called Boundary layer Separation.
- The point on the Solid Body at which the Boundary layer is on the Separation is called point of Separation.

Location of Separation point:

For a given velocity profile, it can be determined whether the Boundary layer has Separated ~~or~~ or on the Verge (Border line) of Separation or it will not Separate from the following Conditions

(i) If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative, then the flow has Separated.

(ii) If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$, then the flow is on the Border line.

(iii) If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is +ve → Not Separated.

10. determine whether the flow has Separated or not, for the following velocity profile

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

→

$$u = \frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\left(\frac{y}{\delta}\right)^3$$

$$u = U\left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]$$

$$\frac{\partial u}{\partial y} = \left[\frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\left(\frac{y}{\delta}\right)^3\right) dy$$

$$\frac{\partial u}{\partial y} = \frac{3U}{2} \cdot \frac{1}{\delta} - \frac{U}{2} \frac{3y^2}{\delta^3}$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{3U}{2\times\delta} - \frac{U}{2}(0)$$

$$= \frac{3U}{2\delta}$$

$$\therefore \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{3U}{2\delta} \; (+ve) \Rightarrow$$ The flow has not separated.

X.

Methods of preventing the Separation of boundary layer.

- ✓ due to Separation of Boundary layer, back flows and edges are formed. and hence Continuous loss of ~~edges~~ energy takes place.
- ✓ So Separation of boundary layer Should be avoided by following methods

(i) Supplying additional enegy from a

(ii) providing a by pass in the slotted wing

(iii) Injecting high velocity fluid in the Boundary layer

(iv) Guidance of flow in a confined passage

(v) Providing a rotating cylinder near the leading edge.

drag force:-

The component of total force in the direction of motion is called drag. This component is denoted by 'FD'

Lift Force:-

The component of the total force in the direction $\perp^r$ to the direction of motion is known as lift